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THE PUPILS' ARITHMETIC

BOOK FOUR



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THE PUPILS' ARITHMETIC

BOOK FOUR

BY

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**MEMBER BOARD OF EXAMINERS, DEPARTMENT OF
EDUCATION, NEW YORK**

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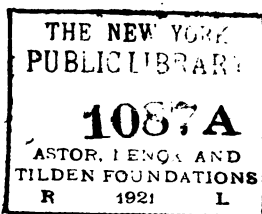
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PREFACE

PART FOUR of *The Pupils' Arithmetic* contains a review of the fundamental rules, a rapid, logical review of common and decimal fractions, a summary of work in denominate numbers, a thorough and comprehensive treatment of percentage, and a brief treatment of simple interest.

In this volume, as in the three books which precede it, the authors have made prominent the following features:

I. A large number of exercises affording abundant practice in the mechanical operations.

II. A large number of simple, practical problems couched in clear terms.

III. Careful grading of the exercises and problems, and the avoidance of long numbers, lengthy processes, and unfamiliar conditions in problem work.

IV. A rapid, comprehensive review of all topics taught in previous grades.

V. The full treatment of each topic in preference to the spiral treatment.

In Part Four the work in common and decimal fractions proceeds with great rapidity. It is de-

revised and corrected, Nov 17/1920.

signed as a review for pupils who have completed the work as treated in Part Three. Any child who cannot proceed with equal rapidity is manifestly not ready for the work in percentage, but requires the slower advance arranged for in Part Three.

The work under denominate numbers is so arranged that *all* tables are presented *for reference only*. The applications have been limited to a very few tables and terms in general use, with valuable drill in money equivalents.

In the treatment of the subject of percentage, great stress has been placed upon its relation to fractions. Applied cases have been freely treated as a part of the larger subject, and not as new topics. A very great number and variety of applications of percentage are presented. Involved and derived cases in connection with taxes, insurance, duties, etc., have been intentionally omitted. There is doubt in the minds of progressive educators as to the value of including such problems anywhere in an elementary school course; there is no doubt, however, as to the claim that they have no place in a book planned for the sixth year.

Simple interest has been included for the benefit of those children who are compelled to leave school before entering the seventh year.

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THE PUPILS' ARITHMETIC
BOOK FOUR



PUPILS' ARITHMETIC

BOOK, IV

REVIEW OF FUNDAMENTAL OPERATIONS

ADDITION

Suggestions to Pupils

1. Add in either direction.
2. Test by adding in the opposite direction.
3. Look for combinations of 10, such as 7 and 3; 6 and 4; 5, 3 and 2; etc.
4. Aim to become rapid in adding correctly. A rate of about 70 figures per minute may easily be attained.

Add :

1.	2.	3.	4.	5.	6.	7.
27	73	62	47	72	68	62
36	84	68	67	75	72	46
94	95	73	58	81	95	82
82	16	79	78	25	32	27
49	27	84	69	37	85	93
76	38	80	89	50	48	15
38	49	95	70	61	73	70
25	50	91	90	25	97	34
19	61	56	81	83	64	82
40	72	52	11	33	73	27
72	83	67	92	98	93	61
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>

8.	9.	10.	11.	12.
5636	9483	8675	29634	36756
8094	6414	9483	84367	95782
4108	9595	3228	40085	76409
4933	8286	5634	36929	50097
8475	4802	2209	71868	94618
6939	7634	5687	46556	93865
5840	9276	7926	59082	70609
6787	2288	9535	32859	93947
4296	7520	6878	76643	85918
<u>7082</u>	<u>9657</u>	<u>2893</u>	<u>69775</u>	<u>48860</u>

13.	14.	15.	16.	17.
71068	68420	99374	31893	53197
3632	5486	28398	25486	91283
85195	71938	56218	17963	46518
27474	21846	70006	2848	29413
83426	7285	39127	39508	36142
12683	39017	86294	563	62911
47197	18892	28511	41794	78305
33641	99375	71619	9865	12916
90759	4680	48305	20907	83155
18413	73897	26500	16848	28006
72787	62848	3571	36979	59136
39540	3096	12846	94865	31520
46851	19764	5189	42973	8275
57689	293	30784	6584	19618
20037	14962	64786	17262	86533
<u>73041</u>	<u>36189</u>	<u>27005</u>	<u>34261</u>	<u>93714</u>

18.	19.	20.	21.	22.
29065	87943	62845	62187	99345
62841	49007	73842	93480	76953
36847	40963	47875	60893	49748
75965	93856	72799	68497	88694
63978	40837	68399	78469	73856
48972	93568	47690	86095	78743
92874	77816	55948	37929	68499
36729	50987	65498	76543	21987
58856	73379	48846	92298	83385
72278	54456	39938	29926	72270
<u>96694</u>	<u>87782</u>	<u>76677</u>	<u>65561</u>	<u>53358</u>

23.	24.	25.	26.	27.
29516	35186	19726	75162	68746
47205	62977	47338	42892	27768
38765	83516	35794	65276	39483
92226	24937	94273	29438	92722
59806	58726	83955	38628	65627
27328	84728	26345	97538	17734
78329	96385	74257	42729	27542
59643	47729	52329	68778	55805
83749	38426	63293	93784	27993
26258	27359	85623	74839	42874
38499	28268	82792	43864	86695
42793	78261	57913	68073	73956
87528	40875	24680	95831	94872
62943	36842	14705	86792	85493
<u>80009</u>	<u>74385</u>	<u>91210</u>	<u>46731</u>	<u>29415</u>

FUNDAMENTAL OPERATIONS

28.	29.	30.	31.	32.
\$ 76.14	\$ 39.47	\$ 295.80	\$ 763.49	\$ 294.08
7.09	116.40	70.10	35.75	26.94
293.17	6.58	400.	92.48	111.60
76.80	216.09	72.18	300.40	25.
315.92	64.82	294.87	63.92	5.40
7.50	4.82	27.92	375.	86.92
200.	17.84	7.92	64.08	7.96
<hr/>				
33.	34.	35.	36.	
\$ 3760.42	\$ 6521.16	\$ 146.18	\$ 4003.17	
193.86	8354.28	3519.27	5116.29	
2007.32	9307.95	7103.52	2814.71	
1476.15	307.28	6917.45	350.18	
8691.28	19.64	2900.63	4192.75	
3582.79	28.15	3174.89	5640.08	
206.05	7315.41	2775.41	2916.70	
2174.18	8596.27	9006.15	1600.05	
1938.15	3514.21	3849.74	3192.18	
9415.74	7608.15	9315.28	8050.39	
<hr/>				

SUBTRACTION

Suggestions to Pupils

2. Test by adding the subtrahend and remainder together. (The sum should be equal to the minuend.) Aim to become rapid in subtracting correctly. A rate of about 50 figures to the minute may easily be attained.

NOTE. — For devices for drills, see Books I, II, and III.

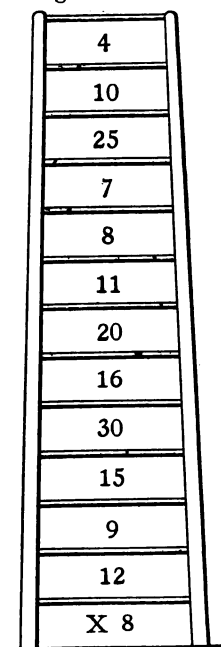
1. $\begin{array}{r} 6387 \\ 2054 \\ \hline \end{array}$	2. $\begin{array}{r} 5690 \\ 2357 \\ \hline \end{array}$	3. $\begin{array}{r} 8205 \\ 4721 \\ \hline \end{array}$
4. $\begin{array}{r} 7800 \\ 3921 \\ \hline \end{array}$	5. $\begin{array}{r} 76843 \\ 27508 \\ \hline \end{array}$	6. $\begin{array}{r} 92005 \\ 39207 \\ \hline \end{array}$
7. $\begin{array}{r} 36849 \\ 16950 \\ \hline \end{array}$	8. $\begin{array}{r} 625846 \\ 78254 \\ \hline \end{array}$	9. $\begin{array}{r} 929007 \\ 169088 \\ \hline \end{array}$
10. $\begin{array}{r} 650000 \\ 543006 \\ \hline \end{array}$	11. $\begin{array}{r} 14628073 \\ 8976829 \\ \hline \end{array}$	12. $\begin{array}{r} 10000 \\ 369 \\ \hline \end{array}$
13. $\begin{array}{r} 72000 \\ 8009 \\ \hline \end{array}$	14. $\begin{array}{r} 609400 \\ 9583 \\ \hline \end{array}$	15. $\begin{array}{r} 111111 \\ 79648 \\ \hline \end{array}$
16. $\begin{array}{r} 628400 \\ 32046 \\ \hline \end{array}$	17. $\begin{array}{r} 10101010 \\ 8546238 \\ \hline \end{array}$	18. $\begin{array}{r} 70006 \\ 8009 \\ \hline \end{array}$
19. $\begin{array}{r} 6342576 \\ 161949 \\ \hline \end{array}$	20. $\begin{array}{r} 56948720 \\ 20750968 \\ \hline \end{array}$	21. $\begin{array}{r} 606060 \\ 77777 \\ \hline \end{array}$

MULTIPLICATION

Suggestions to Pupils

3. 1. Tests. (a) Divide the product by the multiplier; the result should be the multiplicand: (b) divide the product by the multiplicand; the result should be the multiplier.

2. Work long examples in multiplication frequently. Try to reduce the time required until you can multiply at least 40 figures to the minute.



3. When a figure is repeated in a multiplier, do not multiply by that figure a second time, but bring down the former partial product.

ORAL DRILLS

"The Ladder"

4. 1. Begin at the lowest number; go up as quickly as possible. Tell the products.

2. Begin at the highest number; go down as quickly as possible. Tell the products.

3. Begin at the middle and skip about.

4. Write 2, 3, 4, 5, 6, 7, 9, 10, in place of 8. Multiply. Tell the products quickly.

ORAL DRILLS

1. Begin at the right. Multiply each figure in the upper line by each figure in the lower line. Tell the products quickly.

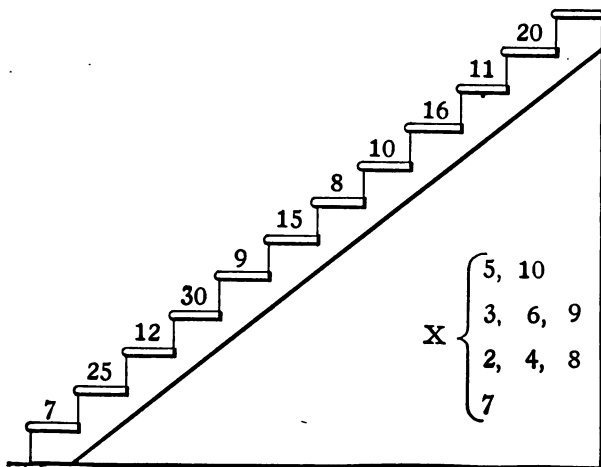
9	30	16	15	12	11	14	10	25	8	20
X 3 5 7 9										

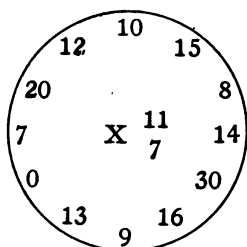
15	9	16	8	12	10	11	15	20	4	30
X 4 6 8 10										

ORAL DRILLS

"The Stairs"

1. Tell the products quickly.

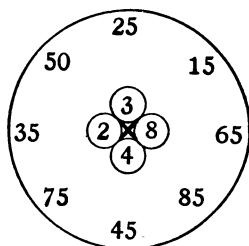
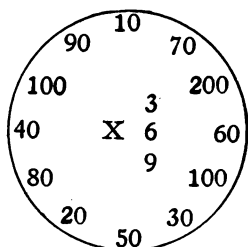




"The Circle"

2. Multiply each number on the circumference by each number in the center of the circle.

Tell the products only.



Some Short Methods in Multiplication

5. 1. To multiply by 10, 100, 1000, etc., rewrite the multiplicand, and annex to the product as many ciphers as there are ciphers in the multiplier.

2. To multiply by 20, 400, 17,000, etc., multiply by all the figures in the multiplier, except the final ciphers, and annex to the product as many ciphers as there are ciphers in the multiplier.

3. To multiply by 50, annex two ciphers to the multiplicand and then divide by 2.

4. To multiply by 25, annex two ciphers to the multiplicand and then divide by 4.

5. To multiply by 125, annex three ciphers to the multiplicand and then divide by 8.

WRITTEN EXERCISES

Find products:

6. 1. $\begin{array}{r} 7894 \\ 628 \\ \hline \end{array}$	2. $\begin{array}{r} 8764 \\ 993 \\ \hline \end{array}$	3. $\begin{array}{r} 6946 \\ 789 \\ \hline \end{array}$	4. $\begin{array}{r} 9285 \\ 637 \\ \hline \end{array}$
--	---	---	---

5. $\begin{array}{r} 7486 \\ 570 \\ \hline \end{array}$	6. $\begin{array}{r} 5789 \\ 690 \\ \hline \end{array}$	7. $\begin{array}{r} 6827 \\ 860 \\ \hline \end{array}$	8. $\begin{array}{r} 7893 \\ 590 \\ \hline \end{array}$
---	---	---	---

9. $\begin{array}{r} 9074 \\ 609 \\ \hline \end{array}$	10. $\begin{array}{r} 8908 \\ 807 \\ \hline \end{array}$	11. $\begin{array}{r} 7095 \\ 508 \\ \hline \end{array}$	12. $\begin{array}{r} 3890 \\ 406 \\ \hline \end{array}$
---	--	--	--

13. $\begin{array}{r} 78964 \\ 5700 \\ \hline \end{array}$	14. $\begin{array}{r} 83945 \\ 6500 \\ \hline \end{array}$	15. $\begin{array}{r} 94652 \\ 8900 \\ \hline \end{array}$
--	--	--

16. $\begin{array}{r} 29643 \\ 5087 \\ \hline \end{array}$	17. $\begin{array}{r} 96408 \\ 3009 \\ \hline \end{array}$	18. $\begin{array}{r} 76008 \\ 3905 \\ \hline \end{array}$
--	--	--

19. $\begin{array}{r} 79648605 \\ 9 \\ \hline \end{array}$	20. $\begin{array}{r} 684907483 \\ 8 \\ \hline \end{array}$
--	---

21-26. Use the same multiplicands as in 19 and 20, and multiply by 7; 13; 17.

27-29. Multiply 72,964 by 25; by 125; by 250; using only the short method.

30-33. Multiply 6864 by 50; by 75; by 500; by 750; using only the short method.

34. Multiply the following by 10; by 100; by 1000: 6384; 7416; 18005.

35. Multiply the following by 20; by 200; by 2000: 564; 2716; 8502.

DIVISION

Suggestions to Pupils

7. 1. Place each quotient figure over the last figure of the corresponding partial dividend.

2. Tests. (a) Multiply the quotient by the divisor and add the remainder; the result should be the dividend: or, (b) divide the dividend by the quotient; the result should be the divisor.

3. If any figure is repeated in the quotient, do not multiply again by that figure, but bring down the former product.

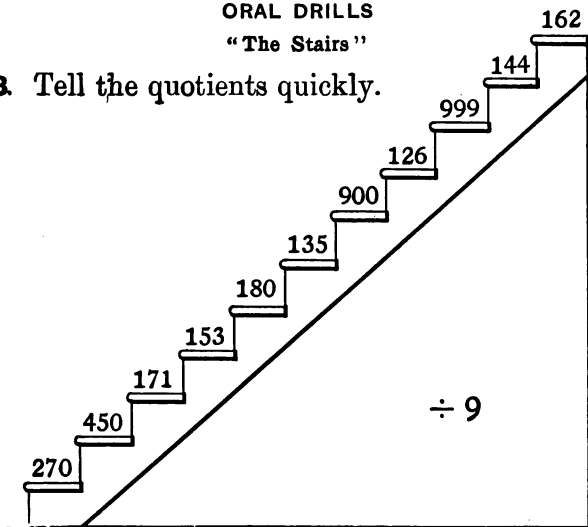
4. If your errors are in subtraction, review and drill that subject before you continue the work in division.

5. Aim to become rapid in dividing correctly.

ORAL DRILLS

"The Stairs"

8. Tell the quotients quickly.

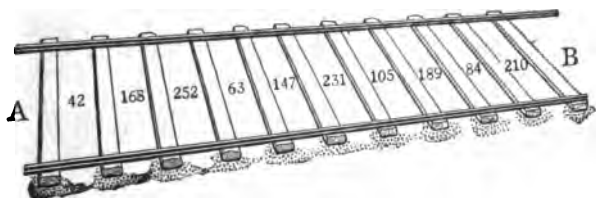


ORAL DRILLS

"Railroad Ties"

Go from *A* to *B* as quickly as possible, and then back from *B* to *A*.

Divide by 7; by 21. .



ORAL DRILLS

"The Ladder"

1. Begin at the lowest number; go up as quickly as possible. Tell the quotients.

2. Begin at the highest number; go down as quickly as possible. Tell the quotients.

3. Begin at the middle and skip about.

4. Make other ladders, writing your own dividends and divisors.

Some Short Methods in Division

9. 1. To divide by 10, 100, 1000, etc., cut off from the right of the dividend by a vertical

112
800
400
104
144
168
128
240
96
162
960
200
÷ 8

line or a decimal point as many figures for the remainder as there are ciphers in the divisor.

2. To divide by 20, 400, 6000, 180,000, etc., cut off from the right of the dividend by a vertical line or a decimal point as many figures as there are ciphers in the divisor; then divide the remaining figures of the dividend by the remaining figures of the divisor.

3. To divide by 50, multiply the dividend by 2 and divide by 100 by pointing off two places from the right of the dividend.

4. To divide by 25, multiply the dividend by 4 and then divide by 100 by pointing off two places from the right of the dividend.

5. To divide by 125, multiply the dividend by 8 and divide by 1000 by pointing off three places from the right of the dividend.

ORAL DRILLS

Divide each number by the number in the rectangle. Tell the quotients instantly.

128	192	48		120	60	135	
90	<div style="border: 1px solid black; padding: 5px; display: inline-block;">16</div>			30	<div style="border: 1px solid black; padding: 5px; display: inline-block;">15</div>		
160	96	64		75	90	165	
42	98	140		40	140	60	
70	<div style="border: 1px solid black; padding: 5px; display: inline-block;">14</div>			80	<div style="border: 1px solid black; padding: 5px; display: inline-block;">20</div>		
168	126	84		120	160	180	

WRITTEN EXERCISES

10. 1-6. Divide 17,846 by 41; by 51; by 61; by 71; by 81; by 91.

7-12. Divide 28,946 by 27; by 47; by 67; by 77; by 87; by 97.

13-18. Divide 97,486 by 321; by 427; by 608; by 723; by 503; by 698.

19-23. Divisor, 16; dividends, 7934; 8927; 6843; 59,436; 72,085. Find quotients.

24-29. Use the dividend 169,403, and divide by every odd number from 17 to 27 inclusive.

30-33. Divide the following numbers by 407: 81,532; 203,687; 59,463; 728,905.

34-38. Use the dividend 260,075, and divide by every even number from 292 to 300 inclusive.

39-41. Divide by 125 (short method only): 628,420; 793,800; 629,570.

42-44. Divide by 25 (short method only): 68,400; 79,630; 829,930.

MISCELLANEOUS ORAL EXERCISES

11. Find the missing numbers:

1. $28 + 30 + ? = 70$

7. $15 + 30 + 30 = ?$

2. $59 - 29 - 9 = ?$

8. $52 + 32 + 12 = ?$

3. $13 + 15 + ? = 34$

9. $47 + 20 + ? = 97$

4. $36 + 24 + 50 = ?$

10. $25 + 35 + ? = 105$

5. $96 - 40 - 50 = ?$

11. $60 + 70 + 80 - 30 = ?$

6. $23 + 28 - 18 = ?$

12. $24 + 34 + 40 = ?$

13. $100 + 40 + 60 - 30 = ?$
14. $250 + 250 - 200 = ?$ 15. $? + 75 + 125 = 300$
16. $? + 80 + 60 + 40 = 220$
17. $22 + 28 + 54 + 36 = ?$
18. $240 + 360 + 500 = ?$ 20. $230 + 230 + 440 = ?$
19. $950 - 400 - 350 = ?$ 21. $450 + 550 + 350 = ?$
22. $1000 - 250 - 500 = ?$
23. $2400 + 3600 + 4000 = ?$
24. $? + 125 + 350 + 500 = 1000$
25. $850 - 400 - 250 - 100 = ?$
26. $126 \times 5 = ?$ 30. $450 \div 30 = ?$
27. $240 \times 3 = ?$ 31. $230 \times 12 = ?$
28. $126 \times 3 = ?$ 32. $1080 \div 12 = ?$
29. $450 \div 15 = ?$ 33. $2835 \div 7 = ?$
34. $134 \times 8 = ?$
35. $(160 \div 4) + (240 \div 3) = ?$
36. $(200 \div 10) - (150 \div 50) = ?$
37. $275 \times 8 = ?$ 44. $3328 \div 16 = ?$
38. $560 \times 12 = ?$ 45. $936 \div 18 = ?$
39. $215 \times 20 = ?$ 46. $1650 \div 20 = ?$
40. $340 \times 200 = ?$ 47. $2025 \div 25 = ?$
41. $3627 \div 9 = ?$ 48. $5800 \div 200 = ?$
42. $1488 \div 12 = ?$ 49. $18,000 \div 3000 = ?$
43. $1575 \div 15 = ?$ 50. $120 \times 5000 = ?$

MISCELLANEOUS ORAL EXERCISES

12. Find the missing numbers :

- | | | |
|-------------------------|--------------------|---------------------|
| 1. $4 \times 125 = 500$ | $8 \times 125 = ?$ | $12 \times 125 = ?$ |
| 2. $5 \times 18 = 90$ | $10 \times 18 = ?$ | $20 \times 18 = ?$ |
| 3. $3 \times 17 = 51$ | $6 \times 17 = ?$ | $9 \times 17 = ?$ |
| 4. $6 \times 21 = 126$ | $12 \times 21 = ?$ | $18 \times 21 = ?$ |
| 5. $10 \times 56 = 560$ | $5 \times 56 = ?$ | $2 \times 56 = ?$ |
| 6. $4 \times 14 = 56$ | $4 \times 28 = ?$ | $8 \times 14 = ?$ |
| 7. $2 \times 27 = 54$ | $4 \times 27 = ?$ | $6 \times 27 = ?$ |
| 8. $12 \times 22 = 264$ | $6 \times 22 = ?$ | $12 \times 44 = ?$ |
| 9. $8 \times 13 = 104$ | $16 \times 13 = ?$ | $4 \times 13 = ?$ |
| 10. $3 \times 31 = 93$ | $6 \times 31 = ?$ | $3 \times 62 = ?$ |
| 11. $125 \div 25 = 5$ | $125 \div 5 = ?$ | $250 \div 5 = ?$ |
| 12. $176 \div 4 = 44$ | $176 \div 8 = ?$ | $176 \div 2 = ?$ |
| 13. $180 \div 12 = 15$ | $180 \div 6 = ?$ | $180 \div 3 = ?$ |
| 14. $240 \div 12 = 20$ | $240 \div 6 = ?$ | $240 \div 2 = ?$ |
| 15. $1200 \div 16 = 75$ | $1200 \div 8 = ?$ | $1200 \div 4 = ?$ |

Find values for

16. $(4 \times 15) + (6 \times 15) = ?$
17. $(3 \times 17) + (7 \times 17) = ?$
18. $(13 \times 12) + (7 \times 12) = ?$
19. $(3 \times 42) + (2 \times 42) = ?$
20. $(50 \times 68) + (50 \times 68) = ?$
21. $(16 \times 21) - (10 \times 21) = ?$

- 22. $(14 \times 21) - (7 \times 21) = ?$
- 23. $(12 \times 20) + (3 \times 20) = ?$
- 24. $(19 \times 12) - (7 \times 12) = ?$
- 25. $(14 \times 9) + (16 \times 9) = ?$
- 26. $(4 \times 15) + (5 \times 18) = ?$
- 27. $(3 \times 22) + (4 \times 8) = ?$
- 28. $(5 \times 16) + (5 \times 20) = ?$
- 29. $(4 \times 21) + (6 \times 10) = ?$
- 30. $(2 \times 35) - (6 \times 5) = ?$
- 31. $(5 \times 40) + (3 \times 25) = ?$
- 32. $(14 \times 10) + (5 \times 12) = ?$
- 33. $(20 \times 13) + (10 \times 4) = ?$
- 34. $(15 \times 8) - (5 \times 10) = ?$
- 35. $(40 \times 17) + (60 \times 17) = ?$

ANALYSIS OF PROBLEMS

Explanations or Forms of Analysis in Multiplication and Division of Whole Numbers

13. 1. A ton of coal weighs 2000 lb. Find the weight of 26 T.

Explanation or Form of Analysis. 26 T. weigh 26 times as much as 1 T.

Therefore, the weight of 26 T. is 26×2000 lb., or 52,000 lb. *Ans.*

2. If a carpenter receives \$22.50 a week, how much money does he receive in 18 weeks?

Explanation or Form of Analysis. The amount received in 18 weeks is 18 times the amount received in 1 week.

Therefore, the amount received in 18 weeks is $18 \times \$22.50$, or \$405 *Ans.*

3. A chair cost \$3.80. Find the cost of 96 chairs at that rate.

Explanation or Form of Analysis. The cost of 96 chairs is 96 times the cost of 1 chair.

Therefore, the cost of 96 chairs is $96 \times \$3.80$, or \$364.80 *Ans.*

4. A storekeeper buys blank books at 40 ct. a dozen and sells them at 76 ct. a dozen. Find his profit on 60 doz. books.

Explanation or Form of Analysis. The profit on 60 doz. books is 60 times the profit on 1 doz. books.

Therefore, the profit on 60 doz. is 60×36 ct., or \$21.60 *Ans.*

5. 34 bbl. of flour weigh 6,664 lb. Find the weight of 1 bbl. of flour.

Explanation or Form of Analysis. The weight of 1 bbl. is $\frac{1}{34}$ of the weight of 34 bbl.

Therefore, the weight of 1 bbl. is $\frac{1}{34}$ of 6664 lb., or $6664 \text{ lb.} \div 34$, or 196 lb. *Ans.* (Compare with No. 1.)

6. A man receives \$450 for working 15 weeks. How much does he receive for working 1 week?

Explanation or Form of Analysis. The amount received for 1 week is $\frac{1}{15}$ of the amount received for 15 weeks.

Therefore, the amount received for 1 week is $\frac{1}{15}$ of \$450, or $\$450 \div 15$, or \$30 *Ans.* (Compare with No. 2.)

7. If a man is paid at the rate of \$30 a week, how many weeks must he work to earn \$480?

Explanation or Form of Analysis. The number of weeks will equal the number of 30's in 480.

Therefore, the number of weeks will equal $480 \div 30$, or 16 *Ans.*

8. If 14 tables cost \$105.28, how much will 1 table cost?

Explanation or Form of Analysis. The cost of 1 table is $\frac{1}{14}$ of the cost of 14 tables.

Therefore, the cost of 1 table is $\frac{1}{14}$ of \$105.28, or $\$105.28 \div 14$, or \$7.52 *Ans.* (Compare with No. 3.)

9. 22 blankets cost \$143. Find the cost of 7 blankets.

Explanation or Form of Analysis. The cost of 1 blanket is $\frac{1}{22}$ of \$143, or \$6.50. (See No. 8.)

The cost of 7 blankets is $7 \times \$6.50$, or \$45.50 *Ans.* (Compare with No. 3.)

10. 24 spoons cost \$138. Find the cost of 12 spoons.

Explanation or Form of Analysis. The cost of 12 spoons is $\frac{1}{2}$ of the cost of 24 spoons.

Therefore, 12 spoons cost $\frac{1}{2}$ of \$138, or \$69 *Ans.*

11. 6 knives cost \$27. Find the cost of 30 knives.

Explanation or Form of Analysis. 30 knives cost 5 times as much as 6 knives. Therefore 30 knives cost $5 \times \$27$, or \$135 *Ans.*

NOTE TO TEACHERS. These forms of analysis are given as suggestions. Any form that is logical and clear may be accepted.

Forms 1, 5, and 7, illustrating respectively the processes of multiplication, partition, and division, are the forms to be memorized if it is desired to memorize any forms of analysis.

Teachers should train pupils to indicate all the operations necessary to the solution of a problem, and to utilize the methods of cancelation and of fractions in performing operations.

PROBLEMS

14. The following problems should be worked mentally, using pencil only for recording answers. When there are several processes in one problem, there is no objection to writing down partial answers; but each operation should be done mentally.

1. A boy who had 120 marbles, won 35 more from one boy and 25 more from another. Afterwards he lost 42 marbles. How many marbles did he then have?

2. A board is 72 in. long. A piece 12 in. long is cut off; the remainder is divided into 3 equal pieces. How long is each piece?

3. After I cut from a board 4 pieces of wood each 10 in. long, there are still 14 in. of wood left. How long was the board at first?

4. A plumber receives \$4.50 a day; his helper receives \$1.50 a day. How much does each receive at the end of four working days?

5. A boy buys 5 lb. of coffee at \$.29 a pound. How much change should he receive from \$2?

6. I bought 2 railroad tickets for \$1.16 each, receiving \$2.68 change. What was the value of the bill that I gave to the ticket agent?

7. An aëroplane flies at the rate of 36 mi. an hour for 2 hr., and at the rate of 25 mi. an hour for 3 hr. How many miles does it cover in the 5 hr.?

8. There are 5280 ft. in a mile. An aëroplane flew 11,000 ft. high. How many feet more than a mile did it fly?

9. How many feet less than 2 mi. did it fly?

10. It requires 16 gal. of gasoline to make a motor go for 5 hr. How much money is spent for gasoline at 22 ct. a gallon in a 10-hr. trip?

11. A train runs from Albany to New York in 3 hr. The distance is 143 mi. At what rate per hour does the train travel?

12. Six rolls of wall paper at 75 ct. a roll were needed for one room, and 9 rolls at \$1.50 a roll, for another room. How much money was spent for the paper for both rooms? How much more was spent for the second room than for the first?

13. An agent sells the following: 26 tickets at 40 ct. each; 15 tickets at 50 ct. each. How much money does he receive?

14. The distance from B to C is 150 mi.; from A to C , 90 mi.; from A to D 180 mi. How far is it from B to A ? From C to D ? From B to D ?



15. The population of a village in 1900 was 2475. The population in 1910 was 6389.

(a) Find the increase.

(b) What was the average annual increase?

16. The population of a town in 1900 was 6890. The population in 1910 was 10,140.

(a) Find the increase in population.

(b) Find the average annual increase.

17. A seamstress receives an order to make 12 aprons, 6 skirts, and 8 gowns. Each apron takes 2 yd. of muslin; each skirt, 5 yd.; and each gown, 4 yd. How many yards of muslin will she require?

18. A dressmaker employs 4 operators, 2 finishers, and 1 draper. The operators receive \$15 each per week; the finishers, \$16 each; and the draper, \$20. How much is paid out weekly in wages?

19. Find the total cost of findings for a dress:

3 whalebones @ 15 ct. each.

2 doz. buttons @ 25 ct. a dozen.

1 belt of tape @ 15 ct.

1 pair of shields @ 25 ct.

3 yd. of cambric @ 20 ct. a yard.

20. A milliner makes a hat at the following cost:

Shape, \$3.00

Velvet for trimming, \$1.50

Wings, \$1.00

She sells the hat for \$8.00. What has she charged for labor and profit?

21. A commission agent agreed to pay a farmer \$1.60 a barrel for his apples. At the end of the season the agent sent the farmer a check for \$54.40. How many barrels of apples had he bought?

22. Find the freight charges on each of the following items:

50 kegs of nails @ 9 ct. a keg.

100 boxes of hammers @ 10 ct. a box.

3 dozen chairs @ 6 ct. each.

Find the total freight charges.

23. Find the freight charges on each of the following:

300 baskets of peaches @ 5 ct. a basket.

200 bbl. of apples @ 8 ct. a barrel.

250 bbl. of potatoes @ 8 ct. a barrel.

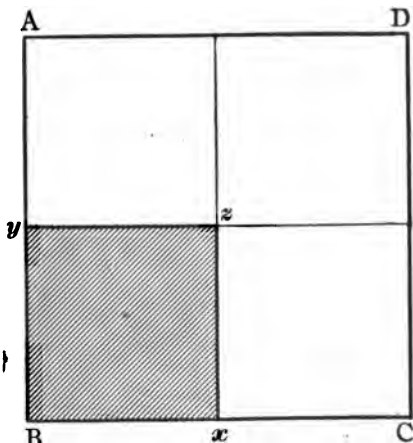
Find the total freight charges.

24. It costs \$1.20 a foot to dig a water well. How deep a well can be dug for \$84?

25. A man bought 250 A. of land @ \$500 an acre. He sold 150 A. @ \$600 an acre, and the remainder @ \$700 an acre. Find his total profit.

26. Strips of wall paper are 18 in. wide. How many strips are needed to paper a wall 24 ft. long?

27. The area of the rectangle $ABCD$ is 144 sq. ft. Bx is $\frac{1}{2}$ of BC . By is $\frac{1}{2}$ of BA . Find the area of the rectangle $Bxzy$.



28. Fifty persons can be comfortably seated in a railroad car.

How many cars are needed to seat 750 persons?

29. A publisher's office mails the following: 100 books, each with a 5-ct. stamp; 250 letters, each with a 2-ct. stamp; 125 circulars, each with a 1-ct. stamp. How much money is spent for postage?

30. A publisher mailed 168 magazines. He arranged them in 12 equal packages. How many magazines in each package?

31. Each package weighed 10 lb. The post-office charges 1 ct. a pound. How much postage was paid on the 12 packages?

32. A boy advertised in a newspaper for a position. His advertisement took 3 lines. The rates were "25 ct. a line, one insertion; 20 ct. a line, each additional insertion." The boy advertised 4 times. How much did he pay?

33. A lady advertised a furnished room to let. The advertisement took 5 lines. The rates were 15 ct. a line for one insertion; 10 ct. a line for each additional insertion. She advertised 5 times. How much did the advertisements cost?

34. A large milk can holds 40 qt. of milk. How many cans are needed for 720 qt. of milk?

35. Thirty dozen eggs are packed in a box. How many boxes are needed for 6300 eggs?

36. Find the cost of 2 pairs of shoes if 6 pairs cost \$27.

37. Find the amount of money earned by a carpenter in 12 days, if he receives \$255 for working 60 days.

38. How far does a train travel in 4 hr., if it travels 486 mi. in 12 hr.?

39. A telephone company received at one station 580 five-cent calls and 260 ten-cent calls in one hour. At that rate, how much money will the company earn in 6 hr.?

40. How many boards, each ten feet long, will be required to make 12 bookshelves, each $2\frac{1}{2}$ ft. long?

41. How much will the wood for the shelves cost at 70 ct. a board?

FACTORING

15. Whole numbers are divided into two classes, **PRIME** and **COMPOSITE**.

NOTE. Prime comes from a word meaning first; composite comes from a word meaning placed together.

A **prime number** is one that can be divided only by itself and 1; as 1, 2, 3, 5, 7, 11, 13, etc.

A **composite number** is one that has factors other than itself and 1; as 4, (2×2); 6, (3×2); 12, (3×4); or ($3 \times 2 \times 2$), etc.

Numbers are divided as **ODD** and **EVEN**.

An **odd number** is one whose unit figure is 1, 3, 5, 7, or 9.

An **even number** is one whose unit figure is 2, 4, 6, 8, or 0.

NOTE. All prime numbers except 2 are odd. Why?

Every composite number can be factored, because it contains at least two exact divisors besides itself and 1.

The whole numbers that, multiplied together produce a number, are called the **factors** of that number.

Taking out equal factors from both dividend and divisor is called **canceling**.

EXERCISE

16. 1. Pick out all prime numbers between 10 and 30.

2. State all the composite numbers between 30 and 50.

3. Name all the odd numbers between 75 and 100.
4. Give all the even numbers from 50 to 75.
5. Name all the odd numbers below 100, that are also prime.

TESTS OF DIVISIBILITY

17. A number is exactly divisible:

By 2, if it ends in 0, 2, 4, 6, 8.

By 3, if the sum of its digits is divisible by 3.

By 4, if the number represented by the two digits at the right is divisible by 4.

By 5, if it ends in 0 or 5.

By 6, if divisible by 2 and 3.

By 8, if the number represented by the three digits at the right is divisible by 8.

By 9, if the sum of its digits is divisible by 9.

By 10, if it ends in 0.

By 12, if it is divisible by 3 and by 4.

EXERCISE

18. Factor:

1. 25, 32, 48, 60, 100, 120, 240.
2. One at a time every composite number from 20 to 30, inclusive.
3. Every composite number from 50 to 60 inclusive.
4. By what numbers is 72 divisible? Why?

CANCELATION

19. 1. Divide $16 \times 27 \times 8$ by $4 \times 20 \times 9$.

Solution

$$\frac{\overset{4}{\cancel{16}} \times \overset{3}{\cancel{27}} \times 8}{\underset{5}{\cancel{4}} \times \cancel{20} \times \cancel{9}} = \frac{24}{5} = 4\frac{4}{5}.$$

Explanation

Take the common factor 4 out of 4 and 16, leaving 1 and 4.

Take 4 out of 20 and 4, leaving 5 and 1.

Take 9 out of 9 and 27.

There remains $\frac{3 \times 8}{5}$ or $\frac{24}{5} = 4\frac{4}{5}$.

EXERCISE

20. Solve the following by cancelation :

1. $24 \times 15 \times 12 \div 9 \times 20 \times 7 = ?$

2. $36 \times 10 \times 13 \div 15 \times 9 \times 11 = ?$

3. $18 \times 8 \times 30 \div 25 \times 6 \times 3 = ?$

4. $7 \times 9 \times 32 \div 22 \times 6 \times 8 = ?$

5. $16 \times 25 \times 17 \div 5 \times 24 \times 10 = ?$

6. $32 \times 35 \times 36 \div 15 \times 20 \times 48 = ?$

7. $\frac{27 \times 12 \times 40 \times 50}{18 \times 26 \times 28 \times 24} = ?$

8. $\frac{25 \times 16 \times 9 \times 10}{10 \times 6 \times 4 \times 5} = ?$

$$9. \frac{28 \times 30 \times 32 \times 40}{24 \times 50 \times 14 \times 16} = ?$$

$$10. \frac{7 \times 8 \times 9 \times 12}{6 \times 3 \times 4 \times 7} = ?$$

$$11. \frac{28 \times 35 \times 42}{6 \times 7 \times 5} = ?$$

$$12. \frac{16 \times 24 \times 30}{10 \times 8 \times 4} = ?$$

$$13. \frac{12 \times 48 \times 24}{16 \times 8 \times 4} = ?$$

$$14. \frac{25 \times 75 \times 50}{10 \times 15 \times 25} = ?$$

15. Divide $21 \times 49 \times 56$ by $7 \times 8 \times 14$.

16. Divide $15 \times 30 \times 60$ by $20 \times 30 \times 15$.

17. Divide $36 \times 60 \times 108$ by $12 \times 48 \times 72$.

18. Divide $26 \times 20 \times 18$ by $30 \times 6 \times 39$.

GREATEST COMMON DIVISOR

21. The greatest common divisor (g. c. d.) of two or more numbers is the greatest number that exactly divides each of them.

EXERCISE

Find the g. c. d. of

1. 72, 84, 96.

4. 16, 24, 32, 40.

2. 60, 40, 120.

5. 36, 54, 72, 180.

3. 13, 39, 52, 65.

6. 45, 60, 75, 90.

LEAST COMMON MULTIPLE

22. The least common multiple (l. c. m.) of two or more numbers is the least number that contains each of them an exact number of times.

EXERCISE

23.

Find the l. c. m. of

- | | |
|----------------|--------------------|
| 1. 12, 18, 24. | 4. 16, 24, 40, 48. |
| 2. 15, 20, 30. | 5. 7, 16, 14, 32. |
| 3. 9, 18, 45. | 6. 9, 12, 18, 27. |

REVIEW OF COMMON FRACTIONS

24. A fraction is one or more of the equal parts of a unit.

A fraction has two terms: the denominator and the numerator.

The **denominator** of a fraction is the number that shows into how many equal parts the unit is divided. It is written below the line.

The **numerator** of a fraction is the number that shows how many of the equal parts are taken to form the fraction. It is written above the line.

Fractions are of two kinds, proper and improper.

A **proper fraction** is a fraction whose numerator is less than its denominator. The value of a proper fraction is less than 1.

$\frac{3}{5}$, $\frac{7}{8}$, $\frac{9}{15}$ are proper fractions.

An **improper fraction** is a fraction whose numerator equals or exceeds its denominator. Its value is equal to 1 or greater than 1.

$\frac{4}{4}$, $\frac{10}{10}$, $\frac{29}{29}$, $\frac{100}{100}$ are improper fractions equal to 1.

$\frac{5}{4}$, $\frac{11}{10}$, $\frac{31}{29}$, $\frac{174}{100}$ are improper fractions whose value is greater than 1.

A **fractional unit** is a fraction whose numerator is 1.

$\frac{1}{4}$, $\frac{1}{29}$, $\frac{1}{7}$, $\frac{1}{63}$ are fractional units.

A **mixed number** is a whole number and a proper fraction written together.

$7\frac{1}{2}$, $9\frac{3}{8}$, $200\frac{4}{5}$, $624\frac{8}{9}$ are mixed numbers.

FORMS OF REDUCTION USED IN COMMON FRACTIONS

25. 1. Changing to higher terms. (Multiply both numerator and denominator by the same number.)

$$\frac{1}{3} = \frac{2}{6}. \quad \frac{5}{8} = \frac{15}{24}. \quad \frac{7}{12} = \frac{35}{60}.$$

2. Changing two or more fractions to their least common denominator.

$$\frac{3}{5} \text{ and } \frac{2}{3} = \frac{9 \text{ and } 10}{15}. \quad \frac{7}{8}, \frac{5}{12} \text{ and } \frac{5}{6} = \frac{21, 10, \text{ and } 20}{24}.$$

3. Changing to lower terms. (Divide both numerator and denominator by the same number.)

$$\frac{8}{12} = \frac{2}{3}. \quad \frac{5}{25} = \frac{1}{5}. \quad \frac{14}{35} = \frac{2}{5}.$$

4. Changing a mixed number or a whole number to an improper fraction.

$$2\frac{1}{2} = \frac{5}{2}. \quad 7\frac{2}{3} = \frac{23}{3}. \quad 6 = \frac{24}{4}.$$

5. Changing an improper fraction to a whole or a mixed number.

$$\frac{9}{9} = 1. \quad \frac{20}{4} = 5. \quad \frac{72}{12} = 6. \quad \frac{15}{4} = 3\frac{3}{4}. \quad \frac{22}{8} = 2\frac{5}{8}. \quad \frac{76}{8} = 9\frac{1}{2}.$$

6. Changing a complex fraction to a simple fraction.

$$\frac{7\frac{1}{2}}{12} = 7\frac{1}{2} \div 12 = \frac{1}{12} \overset{5}{\text{of } \frac{15}{2}} = \frac{5}{8}, \text{ or, } \frac{7\frac{1}{2}}{12} = \frac{15}{24} = \frac{5}{8}.$$

EXERCISE

26. Reduce the following to lowest terms:

- | | | | |
|-----------------------|-----------------------|-----------------------|-----------------------|
| 1. $\frac{4}{12}$ | 2. $\frac{3}{27}$ | 3. $\frac{6}{24}$ | 4. $\frac{5}{25}$ |
| 5. $\frac{9}{21}$ | 6. $\frac{16}{48}$ | 7. $\frac{15}{45}$ | 8. $\frac{12}{36}$ |
| 9. $\frac{12}{30}$ | 10. $\frac{8}{20}$ | 11. $\frac{18}{24}$ | 12. $\frac{14}{21}$ |
| 13. $\frac{20}{28}$ | 14. $\frac{25}{45}$ | 15. $\frac{25}{30}$ | 16. $\frac{12}{32}$ |
| 17. $\frac{18}{54}$ | 18. $\frac{35}{40}$ | 19. $\frac{28}{35}$ | 20. $\frac{24}{72}$ |
| 21. $\frac{24}{60}$ | 22. $\frac{42}{56}$ | 23. $\frac{72}{81}$ | 24. $\frac{120}{140}$ |
| 25. $\frac{25}{55}$ | 26. $\frac{36}{54}$ | 27. $\frac{32}{56}$ | 28. $\frac{28}{49}$ |
| 29. $\frac{44}{77}$ | 30. $\frac{30}{60}$ | 31. $\frac{50}{90}$ | 32. $\frac{240}{320}$ |
| 33. $\frac{144}{240}$ | 34. $\frac{96}{120}$ | 35. $\frac{108}{180}$ | 36. $\frac{90}{270}$ |
| 37. $\frac{50}{400}$ | 38. $\frac{36}{144}$ | 39. $\frac{64}{160}$ | 40. $\frac{81}{360}$ |
| 41. $\frac{56}{140}$ | 42. $\frac{56}{160}$ | 43. $\frac{60}{210}$ | 44. $\frac{80}{360}$ |
| 45. $\frac{100}{120}$ | 46. $\frac{100}{144}$ | 47. $\frac{100}{125}$ | 48. $\frac{100}{150}$ |
| 49. $\frac{75}{125}$ | 50. $\frac{50}{175}$ | 51. $\frac{50}{200}$ | 52. $\frac{50}{125}$ |
| 53. $\frac{40}{200}$ | 54. $\frac{96}{144}$ | 55. $\frac{72}{180}$ | 56. $\frac{72}{144}$ |

EXERCISE 2

Reduce the following to lowest terms:

- | | | | |
|-----------------------|----------------------|-----------------------|-----------------------|
| 1. $\frac{36}{45}$ | 2. $\frac{90}{210}$ | 3. $\frac{45}{75}$ | 4. $\frac{28}{50}$ |
| 5. $\frac{120}{160}$ | 6. $\frac{27}{72}$ | 7. $\frac{40}{48}$ | 8. $\frac{59}{72}$ |
| 9. $\frac{64}{72}$ | 10. $\frac{65}{95}$ | 11. $\frac{45}{120}$ | 12. $\frac{105}{126}$ |
| 13. $\frac{150}{180}$ | 14. $\frac{84}{108}$ | 15. $\frac{120}{135}$ | 16. $\frac{36}{96}$ |
| 17. $\frac{63}{90}$ | 18. $\frac{75}{90}$ | 19. $\frac{60}{160}$ | 20. $\frac{140}{180}$ |
| 21. $\frac{160}{180}$ | 22. $\frac{48}{60}$ | 23. $\frac{175}{225}$ | 24. $\frac{192}{216}$ |

25. $\frac{84}{105}$	26. $\frac{16}{100}$	27. $\frac{120}{180}$	28. $\frac{600}{750}$
29. $\frac{66}{77}$	30. $\frac{28}{42}$	31. $\frac{72}{120}$	32. $\frac{51}{136}$
33. $\frac{38}{57}$	34. $\frac{84}{98}$	35. $\frac{45}{120}$	36. $\frac{108}{180}$
37. $\frac{168}{288}$	38. $\frac{168}{180}$	39. $\frac{95}{247}$	40. $\frac{126}{210}$

27.

EXERCISE

$$1. \frac{3}{4} = \frac{3}{12} = \frac{3}{20} = \frac{3}{16} = \frac{3}{28} = \frac{3}{32} = \frac{3}{40}.$$

$$2. \frac{2}{9} = \frac{2}{18} = \frac{2}{45} = \frac{2}{36} = \frac{2}{54} = \frac{2}{72} = \frac{2}{90}.$$

$$3. \frac{5}{8} = \frac{5}{24} = \frac{5}{80} = \frac{5}{32} = \frac{5}{64} = \frac{5}{96} = \frac{5}{88}.$$

$$4. \frac{4}{5} = \frac{4}{20} = \frac{4}{35} = \frac{4}{50} = \frac{4}{100} = \frac{4}{60} = \frac{4}{?}$$

$$5. \frac{6}{7} = \frac{6}{21} = \frac{6}{42} = \frac{6}{35} = \frac{6}{49} = \frac{6}{70} = \frac{6}{?}$$

6. Change each of the following fractions to thirty-sixths: $\frac{2}{3}$; $\frac{3}{4}$; $\frac{1}{12}$; $\frac{5}{12}$; $\frac{1}{6}$; $\frac{5}{6}$; $\frac{7}{12}$; $\frac{4}{9}$; $\frac{7}{9}$.

7. Change each of the following fractions to seventy-seconds: $\frac{1}{3}$; $\frac{3}{4}$; $\frac{1}{12}$; $\frac{7}{12}$; $\frac{1}{6}$; $\frac{5}{6}$; $\frac{3}{8}$; $\frac{5}{8}$; $\frac{5}{9}$.

Change the following to equivalent fractions having the least common denominator.

$$8. \frac{3}{8} \\ \frac{5}{9} \\ \frac{1}{6} \\ \frac{2}{3}$$

$$9. \frac{1}{2} \\ \frac{5}{8} \\ \frac{2}{9} \\ \frac{3}{4}$$

$$10. \frac{4}{5} \\ \frac{1}{6} \\ \frac{2}{3} \\ \frac{5}{9}$$

$$11. \frac{1}{8} \\ \frac{5}{12} \\ \frac{1}{6} \\ \frac{3}{4}$$

$$12. \frac{5}{6} \\ \frac{3}{5} \\ \frac{7}{12} \\ \frac{2}{3}$$

$$13. \frac{9}{10} \\ \frac{3}{4} \\ \frac{5}{8} \\ \frac{2}{5}$$

$$14. \frac{11}{12} \\ \frac{8}{9} \\ \frac{2}{3} \\ \frac{3}{4}$$

$$15. \frac{7}{12} \\ \frac{5}{12} \\ \frac{5}{6} \\ \frac{3}{8}$$

ADDITION OF FRACTIONS

EXERCISE

28. Find sums :

1. $\frac{1}{3} + \frac{1}{6} + \frac{1}{12}$

2. $\frac{1}{5} + \frac{1}{3} + \frac{1}{10}$

3. $\frac{2}{7} + \frac{4}{9}$

4. $6\frac{1}{2} + 4\frac{3}{4}$

5. $7\frac{3}{8} + 10\frac{3}{4}$

6. $16\frac{2}{3} + 20\frac{1}{4}$

WRITTEN EXERCISES

$$\begin{array}{r} 7. \\ 164\frac{8}{9} \\ 27\frac{5}{12} \\ 600\frac{3}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 8. \\ 728\frac{5}{6} \\ 40\frac{7}{8} \\ 201\frac{1}{3} \\ \hline \end{array}$$

$$\begin{array}{r} 9. \\ 6000\frac{1}{2} \\ 728\frac{5}{9} \\ 46\frac{5}{6} \\ \hline \end{array}$$

$$\begin{array}{r} 10. \\ 643\frac{8}{15} \\ 209\frac{11}{20} \\ 483\frac{11}{12} \\ \hline \end{array}$$

11-14. Make four examples in addition of mixed numbers consisting of 4 addends each, the least common denominators to be, respectively, 36, 48, 72, 120. Find the sum in each example.

SUBTRACTION OF FRACTIONS

ORAL EXERCISES

29. Subtract at sight :

1. From 100 take $8\frac{1}{3}$.

6. $7\frac{1}{2} - 3\frac{1}{4} = ?$

2. From 70 take $12\frac{7}{8}$.

7. $12\frac{3}{8} - 5\frac{1}{4} = ?$

3. Take $18\frac{3}{4}$ from 25.

8. $10\frac{5}{6} - 1\frac{2}{3} = ?$

4. Take $6\frac{7}{11}$ from 22.

9. $9\frac{7}{8} - 3\frac{1}{2} = ?$

5. From 90 take $10\frac{5}{8}$.

10. $8\frac{2}{3} - 1\frac{1}{6} = ?$

$$\begin{array}{r} 11. \\ \text{From } 7\frac{1}{2} \\ \text{take } 6\frac{3}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 12. \\ 12\frac{1}{8} \\ 9\frac{3}{8} \\ \hline \end{array}$$

$$\begin{array}{r} 13. \\ 10\frac{5}{6} \\ 8\frac{2}{3} \\ \hline \end{array}$$

$$\begin{array}{r} 14. \\ 25\frac{1}{5} \\ 17\frac{3}{10} \\ \hline \end{array}$$

Find the difference between

- | | |
|---|---|
| 15. $13\frac{2}{3}$ and $5\frac{1}{4}$. | 16. $16\frac{1}{4}$ and $9\frac{1}{2}$. |
| 17. $18\frac{1}{6}$ and $11\frac{1}{3}$. | 18. $20\frac{3}{4}$ and $12\frac{7}{8}$. |
| 19. $60\frac{1}{5}$ and $21\frac{3}{4}$. | 20. $30\frac{2}{9}$ and $11\frac{5}{6}$. |

WRITTEN EXERCISES

- | | 1. | 2. | 3. |
|----------|--------------------|-------------------|--------------------|
| 30. From | $792\frac{5}{8}$ | $2000\frac{1}{6}$ | $7209\frac{5}{12}$ |
| take | $167\frac{11}{12}$ | $728\frac{3}{4}$ | $846\frac{7}{8}$ |

Prove examples 2 and 3.

- | | 4. | 5. | 6. |
|------|--------------------|--------------------|-------------------|
| From | $2094\frac{7}{15}$ | $29643\frac{1}{9}$ | $1000\frac{5}{8}$ |
| take | $1888\frac{9}{10}$ | $7689\frac{1}{4}$ | $761\frac{9}{24}$ |

7-12. Make six examples in subtraction of mixed numbers, the least common denominators to be, respectively, 27, 36, 42, 64, 80, 144.

Find the remainder in each example.

MULTIPLICATION OF FRACTIONS

31. 1. To multiply a fraction by a whole number.

(a) $\frac{5}{8}$ multiplied by 12. (b) $\frac{5}{8}$ multiplied by 9.

$$\frac{5}{8} \times \overset{3}{12} = \frac{15}{2} = 7\frac{1}{2}.$$

$$\frac{5}{8} \times 9 = \frac{45}{8} = 5\frac{5}{8}.$$

NOTE. Cancel, wherever possible.

2. To multiply a whole number by a fraction.

(a) 12 multiplied by $\frac{5}{8}$ or $\frac{5}{8}$ of 12.

(b) 9 multiplied by $\frac{5}{8}$ or $\frac{5}{8}$ of 9.

$$\frac{5}{8} \text{ of } 12\frac{3}{2} = \frac{15}{2} = 7\frac{1}{2}; \text{ or } 12\frac{3}{2} \times \frac{5}{8} = \frac{15}{2} = 7\frac{1}{2}$$

$$\frac{5}{8} \text{ of } 9 = \frac{45}{8} = 5\frac{5}{8}; \text{ or } 9 \times \frac{5}{8} = \frac{45}{8} = 5\frac{5}{8}$$

3. To multiply a fraction by a fraction.

$$\frac{7}{8} \times \frac{4}{5} \text{ or } \frac{4}{5} \text{ of } \frac{7}{8}.$$

$$\frac{7}{8} \times \frac{4}{5} = \frac{7}{10}; \text{ or } \frac{4}{5} \text{ of } \frac{7}{8} = \frac{7}{10}.$$

4. To multiply a mixed number by a whole number. $3\frac{4}{5}$ multiplied by 10.

$$\begin{array}{r} 3\frac{4}{5} \\ 10 \\ \hline 30 \\ 38 \end{array}$$

5. To multiply a whole number by a mixed number.

24 multiplied by $8\frac{5}{6}$.

$$\begin{array}{r} 24 \\ 8\frac{5}{6} \\ \hline 20 \\ 192 \\ 212 \end{array}$$

Explanation. $\frac{1}{6}$ of 24 = 4. $\frac{5}{6} = 20$.
8 times 24 = 192.
192 + 20 = 212.

NOTE. When the denominator is not an exact divisor of the multiplicand, multiply by the numerator first and then divide by the denominator.

$$\begin{array}{r} 28 \times 6\frac{3}{4} \\ 6\frac{3}{4} \\ 5 \overline{) 84} \\ 16\frac{3}{4} \\ 168 \\ \hline 184\frac{3}{4} \end{array}$$

6. To multiply a mixed number by a fraction or a fraction by a mixed number.

$$\frac{7}{8} \times 20\frac{2}{3}, \text{ or } \frac{7}{8} \text{ of } 20\frac{2}{3}.$$

$$\begin{array}{r} 31 \\ 7 \\ 8 \\ 4 \end{array} \times \frac{\cancel{62}}{3} = \frac{217}{12} = 18\frac{1}{12}.$$

7. To multiply a mixed number by a mixed number.

$$18\frac{3}{4} \times 12\frac{2}{5}.$$

$$\begin{array}{r} 15 \\ 75 \\ 4 \\ 2 \end{array} \times \begin{array}{r} 31 \\ \cancel{62} \\ 5 \end{array} = \frac{465}{2} = 232\frac{1}{2}.$$

32. GENERAL RULE FOR MULTIPLICATION OF FRACTIONS. A. To multiply a whole number by a fraction, or a fraction by a whole number, multiply the numerator and the whole number together and divide by the denominator. Cancel wherever possible.

B. To multiply fractions and mixed numbers, change all mixed numbers to improper fractions; multiply the numerators for a new numerator, and the denominators for a new denominator. Cancel wherever possible.

ORAL EXERCISES

33. Multiply at sight:

1. $\frac{7}{8}$ by 8; by 10; by 12; by 16.
2. $\frac{5}{6}$ by 6; by 12; by 18; by 24.
3. $\frac{2}{3}$ by 4; by 7; by 8; by 11.
4. $\frac{4}{9}$ of 3 = ? $\frac{4}{9}$ of 6 = ? $\frac{4}{9}$ of 12 = ? $\frac{4}{9}$ of 18 = ?

Find products:

5.	$6 \times \frac{2}{3}$	$6 \times \frac{5}{6}$	$6 \times \frac{3}{8}$
6.	$8 \times \frac{3}{4}$	$8 \times \frac{5}{8}$	$8 \times \frac{5}{2}$
7.	$5 \times \frac{1}{6}$	$5 \times \frac{3}{4}$	$5 \times \frac{7}{2}$
8.	$10 \times \frac{1}{4}$	$10 \times \frac{1}{6}$	$10 \times \frac{3}{8}$
9.	$12 \times \frac{1}{3}$	$12 \times \frac{7}{6}$	$12 \times \frac{9}{8}$
10.	$10 \times 1\frac{1}{2}$	$10 \times 2\frac{1}{5}$	$10 \times 1\frac{4}{5}$
11.	$20 \times 1\frac{1}{4}$	$20 \times 2\frac{1}{2}$	$20 \times 2\frac{3}{5}$
12.	$50 \times 1\frac{1}{10}$	$50 \times 2\frac{1}{2}$	$50 \times 4\frac{2}{5}$

WRITTEN EXERCISES

34. Multiply:

- $\frac{5}{9}$ by 21; $\frac{7}{8}$ by 18; $\frac{9}{10}$ by 26; $\frac{5}{6}$ by 32.
- $\frac{4}{5}$ by 23; $\frac{6}{7}$ by 15; $\frac{5}{9}$ by 30; $\frac{3}{7}$ by 42.
- 13 by $\frac{5}{8}$; 12 by $\frac{6}{7}$; 21 by $\frac{5}{6}$; $\frac{7}{8}$ by 62.
- 20 by $1\frac{1}{2}$; 24 by $1\frac{3}{5}$; 29 by $1\frac{7}{5}$.
- $\frac{3}{5}$ of 125 = ? $\frac{5}{8}$ of 74 = ? $\frac{7}{9}$ of 83 = ?
- $\frac{2}{5}$ of \$165 = ? $\frac{3}{8}$ of \$120 = ? $\frac{9}{10}$ of \$250 = ?
- $\frac{7}{12}$ of $12\frac{4}{5}$ = ? $\frac{5}{6}$ of $13\frac{1}{5}$ = ? $\frac{8}{11}$ of $18\frac{1}{3}$ = ?
- $12 \times 8\frac{3}{4}$ = ? $160 \times 19\frac{2}{3}$ = ? $75 \times 62\frac{1}{2}$ = ?
- $42\frac{1}{2} \times 8\frac{3}{5}$ = ? $17\frac{1}{2} \times 16\frac{2}{3}$ = ? $37\frac{4}{5} \times 18\frac{3}{4}$ = ?
- $1,872 \times 90\frac{3}{4}$ = ? $1,600 \times 8\frac{5}{9}$ = ? $7,000 \times 8\frac{3}{7}$ = ?
- $\frac{7}{8}$ of $\frac{4}{11}$ of 40 = ? $\frac{5}{9}$ of $\frac{7}{10}$ of 200 = ?

DIVISION OF FRACTIONS

35. 1. To divide a fraction by a whole number.

$$\frac{7}{8} \div 3$$

$$\frac{1}{3} \text{ of } \frac{7}{8} = \frac{7}{24}.$$

2. To divide a mixed number by a whole number.

(a) When the mixed number is less than the divisor.

(b) When the mixed number is greater than the divisor.

$$(a) \quad 5\frac{2}{3} \div 8 \qquad \frac{1}{8} \text{ of } 1\frac{7}{8} = 1\frac{1}{4}.$$

$$(b) \quad 431\frac{2}{3} \div 8$$

$$8 \overline{)431\frac{2}{3}} \quad \left(\frac{7\frac{2}{3}}{8} \right) = 53\frac{2}{3} \text{ Ans.}$$

3. To divide a whole number by a fraction.

$$12 \div \frac{5}{6}$$

$$12 \times \frac{6}{5} = \frac{72}{5} = 14\frac{2}{5}$$

4. To divide a whole number by a mixed number.

$$75 \div 8\frac{3}{4}$$

$$\begin{array}{r} 15 \\ 75 \end{array} \times \frac{4}{\frac{35}{7}} = \frac{60}{7} = 8\frac{4}{7}$$

5. To divide a fraction by a fraction.

$$\frac{8}{11} \div \frac{4}{5}$$

$$\frac{2}{11} \times \frac{5}{4} = \frac{10}{11}$$

6. To divide a mixed number by a mixed number.

$$18\frac{3}{4} \div 12\frac{1}{2}$$

$$\frac{75}{4} \times \frac{2}{25} = \frac{3}{2} = 1\frac{1}{2}$$

36. GENERAL RULE FOR DIVISION OF FRACTIONS. To divide one fraction or mixed number by another, invert the divisor, and then proceed as in multiplication of fractions.

ORAL EXERCISES

37. Divide:

1. $\frac{7}{12}$ by 3; by 6; by 7; by 8.
2. $\frac{8}{15}$ by 4; by 2; by 8; by 10.
3. $\frac{15}{16}$ by 5; by 3; by 10; by 6.
4. $2\frac{1}{2}$ by 2; by 5; by 4; by 6.
5. $18\frac{3}{4}$ by 5; by 15; by 6; by 10.
6. 20 by $1\frac{1}{2}$; by $2\frac{1}{2}$; by $3\frac{1}{2}$; by $4\frac{1}{2}$.
7. 60 by $3\frac{1}{3}$; by $7\frac{1}{2}$; by $6\frac{2}{3}$; by $3\frac{3}{4}$.
8. $\frac{7}{8}$ by $\frac{5}{16}$; by $\frac{1}{4}$; by $\frac{3}{4}$; by $\frac{1}{7}$.
9. $12\frac{1}{2}$ by $1\frac{1}{3}$; by $2\frac{1}{2}$; by $1\frac{1}{4}$; by $7\frac{1}{2}$.
10. $62\frac{1}{2}$ by $2\frac{1}{2}$; by $12\frac{1}{2}$; by $6\frac{1}{4}$; by $3\frac{1}{3}$.

WRITTEN EXERCISES

38. Find the quotients:

1. $4,692\frac{5}{8} \div 9$; by 7; by 6; by 10.
2. $643\frac{2}{3} \div \frac{7}{8}$; by $\frac{5}{6}$; by $\frac{3}{8}$; by $\frac{6}{7}$.

3. $43\frac{3}{4} \div 2\frac{1}{2}$; by $6\frac{2}{3}$; by $2\frac{4}{5}$; by $8\frac{3}{4}$.

4. $7\frac{2}{3} \div 2\frac{7}{8}$; by $5\frac{1}{2}$; by $\frac{9}{23}$; by $1\frac{1}{2}$.

5. $48\frac{4}{5} \div 8\frac{3}{4}$; by $7\frac{1}{2}$; by $12\frac{1}{5}$; by $1\frac{2}{3}$.

6. The divisor is 18; the dividend is $265\frac{1}{8}$.
Quotient?

7. The divisor is $6\frac{2}{3}$; the dividend is 1,000.
Quotient?

8. The divisor is $7\frac{1}{2}$; the dividend is 5.
Quotient?

9. The divisor is 50; the dividend is $269\frac{1}{11}$.
Quotient?

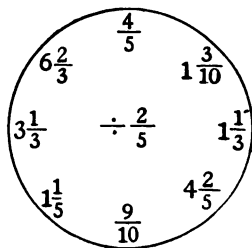
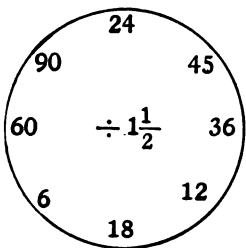
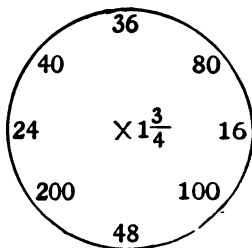
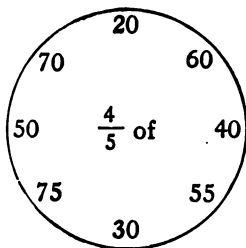
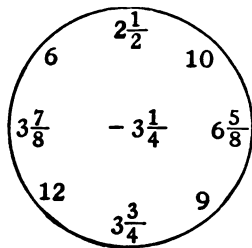
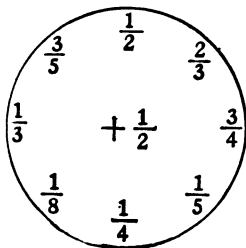
10. The divisor is $\frac{1}{5}$; the dividend is $96\frac{3}{8}$.
Quotient?

. Find the missing term:

	DIVIDEND	DIVISOR	QUOTIENT
11.	$43\frac{3}{4}$	10	?
12.	144	?	$7\frac{1}{2}$
13.	$37\frac{1}{2}$	$3\frac{3}{4}$?
14.	$\frac{7}{8}$?	$\frac{11}{16}$
15.	200	$18\frac{3}{4}$?

ORAL DRILLS

"The Circle"



TYPE PROBLEMS INVOLVING FRACTIONAL RELATIONS OF NUMBERS

First Type Problem: To find a part of a number

39. A man earns 18 dollars a week. What does he earn in $\frac{2}{3}$ of a week?

Explanation. The amount earned in $\frac{2}{3}$ of a week is $\frac{2}{3}$ of the amount earned in a week. Therefore the amount is $\frac{2}{3}$ of \$18 = \$12 *Ans.*

Second Type Problem : To find what fraction one number is of another number

A farmer had 60 sheep. He sold 15 sheep. What part of his flock did he sell?

Explanation. The part sold is 15 sheep out of a flock of 60 sheep, or $\frac{15}{60}$ of the flock. Therefore the part sold is $\frac{15}{60} = \frac{1}{4}$ *Ans.*

Third Type Problem : Finding the whole when a fractional part of it is given

First Case

(A) To find a number when a fractional part of it is given.

$\frac{3}{5}$ of Mary's age is 12 years. How old is Mary?

Explanation. $\frac{3}{5}$ of Mary's age is 12 years. Therefore $\frac{1}{5}$ of her age equals $\frac{1}{3}$ of 12. Her full age equals

5 times $\left(\frac{1}{3} \text{ of } 12\right)$ or $\frac{5}{3}$ of $12 = 20$ yr. *Ans.*

A shorter explanation is: $\frac{3}{5}$ of her age equals 12; then her full age equals $\frac{5}{3}$ of 12 = 20 yr. *Ans.*

Second Case

(B) To find a number when the number minus a fractional part is given.

A boy loses $\frac{1}{3}$ of his marbles. He then has 30 marbles. How many marbles had he at first?

Explanation. Since he lost $\frac{1}{3}$ of his marbles, 30 marbles is $\frac{2}{3}$ of the original number he had.

$\frac{2}{3}$ of the original number = 30 marbles.

$\frac{1}{3}$ of the original number = $\frac{1}{2}$ of 30 = 15 marbles.

$\frac{2}{3}$ of the original number = $3 \times 15 = 45$ marbles.

Or, a shorter explanation is:

$\frac{2}{3}$ of the number of marbles = 30.

All the marbles = $\frac{3}{2}$ of 30 = 45 *Ans.*

Third Case

(C) To find a number, when the number plus a fractional part is given.

A boy added to his savings $\frac{2}{5}$ of the original amount. He then had \$21 in his bank. What was the original amount of his savings?

Explanation. Since he added $\frac{2}{5}$ to the original $\frac{3}{5}$, his savings are $\frac{7}{5}$ of the original amount.

$\frac{7}{5}$ of the original amount = \$21.

$\frac{1}{5}$ of the original amount = ($\frac{1}{7}$ of \$21) = \$3.

$\frac{3}{5}$ of the original amount = $5 \times 3 = \$15$ *Ans.*

Or, a shorter explanation is:

$\frac{7}{5}$ of the original amount = \$21.

The original amount equals $\frac{5}{7}$ of \$21 = \$15 *Ans.*

ORAL PROBLEMS

40. 1. A clerk sells goods for the following amounts: $\$12\frac{1}{2}$, $\$8\frac{1}{4}$, $\$6\frac{3}{4}$, $\$10$. Find the amount of the four sales.

2. Harry had $\$21\frac{1}{2}$. He spent $\$1\frac{4}{5}$. How much money did he have left?

3. A butcher sells $9\frac{1}{2}$ lb. of meat; $6\frac{1}{4}$ lb.; $8\frac{3}{4}$ lb.; $5\frac{1}{2}$ lb. What are the total sales?

4. How many pencils are in $\frac{1}{3}$ of a dozen? in $\frac{3}{4}$ of a dozen?

5. Since 2 pints make a quart, what part of a quart is a pint?

6. A piece of wood $1\frac{7}{8}$ ft. long was cut from a plank 6 ft. long. What was the length of the remaining piece?

7. If $2\frac{1}{4}$ lb. of flour are taken from a 5-lb. bag, what is the weight of the flour still in the bag?

8. How much must be added to $3\frac{3}{8}$ to make $6\frac{1}{8}$?

9. A book worth 80 ct. is sold for $\frac{3}{5}$ of its value. For how much is it sold?

10. How many minutes are there in $\frac{2}{3}$ of an hour?

11. How much longer (expressed in minutes) is $\frac{3}{4}$ of an hour than $\frac{2}{3}$ of an hour?

12. A grocer buys 64 bbl. of apples and finds that $\frac{1}{8}$ of them are bad. How many barrels can he sell?

13. A piece of glass, measuring 48 sq. in., is cut into 4 equal pieces. How many square inches are there in the surface of each piece?

14. From a piece of cloth measuring 20 yd., $6\frac{1}{4}$ yd. are sold at \$2 a yard. How much money was received?

15. The rest of the cloth is sold for \$1 a yard. How much money is received for it?

16. A wall is 30 ft. long. How many strips of paper each $1\frac{1}{2}$ ft. wide will be needed to paper the wall?

17. How much greater is $\frac{3}{4}$ of 28 than $\frac{5}{6}$ of 24?

18. How much less is $\frac{1}{2}$ of 16 than $\frac{3}{4}$ of 20?

19. How much greater is $\frac{7}{8}$ of 56 than $\frac{3}{4}$ of 56?

20. How much less is $\frac{1}{2}$ of 90 than $\frac{5}{6}$ of 90?

21. How much money is saved by buying 10 gal. of oil at 24 ct. a gallon, rather than by buying it at 8 ct. a quart?

22. One half of a man's salary is \$18. Find $\frac{1}{4}$ of his salary.

23. The distance between two cities is 270 miles. What is $\frac{2}{3}$ of the distance?

24. If $\frac{2}{3}$ of the distance between two cities is 150 miles, what is the entire distance between them?

25. If my monthly salary is \$240, how much do I earn in $\frac{3}{4}$ of a month?

26. If I earn \$55 in $\frac{1}{4}$ of a month, how much do I earn in 1 mo.?

27. A grocer divides 192 lb. of flour equally into 12 packages. How many pounds are in each package?

28. If $\frac{1}{12}$ of the amount of flour in a barrel is $16\frac{1}{3}$ lb., how many pounds are there in the barrel?

29. The duty on a vase worth \$560 is $\frac{2}{5}$ of its value. Find the amount of duty.

30. If the duty on a vase is \$85 and amounts to $\frac{1}{5}$ of its value, what is the value of the vase?

31. A runner can do a 100-yard dash in 10 seconds. How many yards can he do in 1 second?

32. At \$2 $\frac{1}{4}$ a day, how much will a man earn in $6\frac{1}{2}$ days?

33. If a man earns \$10 $\frac{1}{4}$ in $4\frac{1}{2}$ days, how much does he earn in 1 day?

34. A tub of butter contains 53 lb. From it the following quantities were sold: $\frac{1}{2}$ lb., $4\frac{1}{2}$ lb., $1\frac{1}{4}$ lb., $6\frac{1}{2}$ lb., 8 lb. How many pounds were sold? How many pounds remained in the tub?

35. At \$ $\frac{2}{5}$ a pound, how much will 7 lb. of butter cost? How much change should I receive from \$3?

36. A hook $5\frac{1}{2}$ in. long is driven $3\frac{3}{4}$ in. into a wall. How much of the hook projects from the wall?

37. Find the cost of $8\frac{1}{2}$ lb. of fish at 12 ct. a pound.

38. Find the cost of $6\frac{1}{2}$ doz. eggs at 32 ct. a dozen.

39. $\frac{1}{4}$ lb. of meat costs 6 ct. Find the cost of $\frac{3}{4}$ lb.

40. $\frac{1}{4}$ lb. of meat costs 6 ct. How much meat may be bought for 30 ct.?

41. $\frac{1}{2}$ yd. of ribbon costs 10 ct. Find the cost of $1\frac{1}{2}$ yd.

42. $\frac{1}{2}$ yd. of ribbon costs 10 ct. How much ribbon may be bought for 90 ct.?

43. A boy earns $\$ \frac{3}{5}$ a day. In how many days will he earn $\$ 1\frac{1}{5}$? $\$ 2\frac{2}{5}$?

44. 4 yd. of cloth costs \$9. How much do 8 yd. cost?

45. 3 lb. of tea cost $\$ 1\frac{1}{4}$. Find the cost of 6 lb.

46. I spend 30 ct. for candies at 2 for 1 ct. How many candies should I receive for the same money, if they cost 3 for 1 ct.?

47. $30\frac{1}{2}$ T. of coal were divided among a number of families. Each family received $\frac{1}{2}$ T. How many families were supplied.

WRITTEN PROBLEMS

41. 1. A man who is walking from Albany to New York covers the following distances: $15\frac{3}{4}$ mi., $17\frac{5}{8}$ mi., $19\frac{1}{8}$ mi., $14\frac{1}{2}$ mi. How far has he walked?

2. A sailing vessel has sailed $9\frac{3}{8}$ mi., $10\frac{7}{8}$ mi., $7\frac{5}{8}$ mi., $8\frac{1}{4}$ mi., and $6\frac{1}{2}$ mi. How much farther must it sail to complete a trip of $50\frac{1}{2}$ mi.?

3. The cook in a restaurant uses the following amounts of butter: $8\frac{1}{4}$ lb., $12\frac{1}{2}$ lb., $10\frac{1}{8}$ lb., $7\frac{7}{8}$ lb., $6\frac{3}{4}$ lb. What is the total amount of butter used?

4. If the butter costs $31\frac{1}{2}$ ct. a pound, what is its total cost?

5. 8 bbl. flour weighing 196 lb. each were bought at the rate of $4\frac{1}{4}$ ct. a pound and sold at the rate of $5\frac{1}{2}$ ct. a pound. Find the profit.

6. An express company delivered 76 packages, each weighing $6\frac{1}{4}$ lb. What was the total weight?

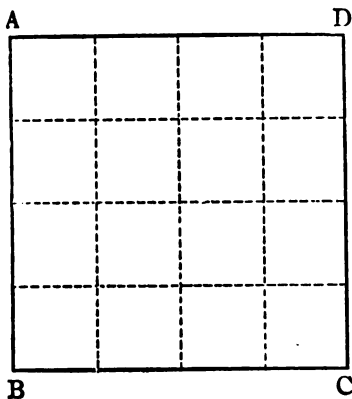
7. If the company charged $\frac{3}{4}$ ct. a pound for the delivery, how much money did it receive for delivering all the packages?

8. From a piece of cloth containing $50\frac{1}{2}$ yd., the following pieces were cut: $6\frac{5}{8}$ yd., $2\frac{1}{4}$ yd., $7\frac{3}{4}$ yd., 20 yd., $5\frac{3}{4}$ yd. How many yards remained?

9. What is the value of the remaining piece if sold at $\$2\frac{1}{4}$ a yard?

10. $\frac{3}{4}$ of the number of boys in a class is 33. How many boys are in the class?

11. A board was divided into 2 equal parts. Then each part was subdivided into 8 equal parts. If one of the small pieces was $2\frac{1}{4}$ in. long, how long was the board originally?



12. $ABCD$ is a square. Each side is 48 in. long. The square is divided into 16 equal squares. How long is a side of one of the smaller squares?

13. If a man earns \$ $3\frac{1}{4}$ a day and spends \$ $1\frac{1}{2}$ a day, how much will he save in 30 days?

14. \$15,000 is divided into 12 equal parts. William gets 3 of the parts. How much money does William get?

15. $\frac{1}{5}$ of a number is $92\frac{1}{4}$. How much is $\frac{1}{10}$ of that number?

16. A train which has been going at the rate of $16\frac{1}{2}$ mi. an hour increases its speed by $\frac{3}{8}$ of its former rate. At what rate per hour does it travel after the increase?

17. A train has a distance of 244 mi. to go. It travels at the rate of 32 mi. an hour for $3\frac{1}{2}$ hr. Then it decreases its speed by $\frac{1}{4}$ of its former rate. How long will it take the train to go the rest of the journey?

18. If a man sells a horse that cost him \$420, at a gain of $\frac{3}{8}$ of the cost, how much does he receive for the horse? How much does he gain?

19. In a town containing 19,000 people, 3 out of every 5 were born in a foreign country. How many were of foreign birth?

20. A school contains 2448 boys. 5 out of every 8 were born in New York. How many boys were born in New York?

21. Mr. Jones wants to sell 16 horses at \$850 each. He offers Mr. Smith a commission of $\frac{1}{6}$ of the value of the horses for selling them. What amount of commission will Mr. Smith earn, if he sells $\frac{3}{4}$ of the horses?

22. An agent agrees to take all the apples from Mr. Gray's orchard, at the rate of $\$1\frac{1}{5}$ per barrel. Mr. Gray sends the agent 750 bbl. If the agent pays $\$ \frac{3}{10}$ per barrel for freight, how much do the apples cost him?

23. For how much per barrel must the agent sell the apples to gain $\$ \frac{3}{4}$ on each barrel?

24. A magazine publisher mails 144 packages of magazines. Each package contains 16 magazines. Each magazine weighs $1\frac{1}{4}$ lb. The post office charges 1 ct. a pound for postage. How much money must be paid for postage on the magazines?

25. How many packages are needed to hold 81 lb. of morning-glory seeds, if each package holds $\frac{3}{4}$ of a pound?

26. How much will 96 lb. of seeds cost at 16 ct. a package, if each package contains $\frac{3}{8}$ lb.?

27. A wheel makes $30\frac{2}{7}$ revolutions a minute. How many revolutions will it make in $12\frac{1}{4}$ min.?

28. A theater agent sold the following tickets:

850 at \$2;

700 at $\$1\frac{1}{2}$;

920 at $\$1\frac{1}{4}$.

Find the total receipts.

29. How many cans each containing $\frac{1}{4}$ of a quart can be filled from 620 qt.?

30. A pump pours $120\frac{1}{2}$ gal. of water into a tank every minute. How long will it take to fill a tank containing $843\frac{1}{2}$ gal.?

31. A cubic foot of water weighs $62\frac{1}{2}$ lb. What is the weight of the water in a tank that contains $55\frac{1}{5}$ cu. ft.?

32. How much profit is made by buying 300 lb. of starch at the rate of $4\frac{1}{2}$ ct. per pound and selling it at the rate of 3 lb. for 17 ct.?

33. If the aggregate weight of 65 boys is $5557\frac{1}{2}$ lb., what is the average weight of each boy?

34. The cargo of a boat consisted of

24 bales weighing $112\frac{1}{2}$ lb. each;

32 boxes weighing $14\frac{1}{2}$ lb. each;

50 bbl. weighing $120\frac{1}{4}$ lb. each.

Find the total weight of the cargo.

35. A tank contained 1500 gal. of water. Then $220\frac{1}{2}$ gal., $315\frac{1}{4}$ gal., and 750 gal. were drawn off.

Later $325\frac{1}{4}$ gal. were poured in. How many gallons were there then in the tank?

VOTE POLLED IN CERTAIN COUNTIES IN NEW YORK
STATE. FIND VALUES FOR ?

	COUNTY	VOTE IN 1910	VOTE IN 1908	FRACTIONAL INCREASES IN 1910 OVER 1908	FRACTIONAL DECREASES IN 1910 OVER 1908
36.	Albany . .	20,500	20,000	?	
37.	Broome . .	6,500	10,000	?
38.	Clinton . .	4,400	5,400	?
39.	Delaware . .	4,200	7,300	?
40.	Erie . . .	50,000	37,500	?	
41.	Hamilton . .	420	580	?
42.	Nassau . .	5,400	5,600	?
43.	Niagara . .	5,200	9,500	?
44.	Orange . .	11,400	10,000	?	
45.	Putnam . .	1,700	1,500	?	
46.	Saratoga . .	7,100	7,600	?

47. A girl is employed to address envelopes. She receives $\$ \frac{3}{4}$ for every 1000 envelopes she addresses. How much will she receive for addressing 12,500 envelopes?

48. In a large business there are the following employees: 250 persons at $\$ 6\frac{1}{2}$ a week;
110 persons at $\$ 10\frac{1}{2}$ a week;
80 persons at $\$ 15\frac{1}{2}$ a week;

How much money is paid in wages each week?

Find the cost in each of the following:

49. 128 bu. wheat at $90\frac{3}{8}$ ct. a bushel.

50. 258 bu. corn at $34\frac{1}{4}$ ct. a bushel.

51. 640 bu. oats at $35\frac{1}{2}$ ct. a bushel.

52. 459 bbl. pork at $\$16\frac{1}{4}$ a barrel.

53. 750 lb. lard at $\$10\frac{1}{2}$ for each 100 lb.

54. 860 lb. turkeys at $20\frac{1}{4}$ ct. a pound.

55. 620 lb. chickens at $18\frac{3}{4}$ ct. a pound.

56. 460 lb. ducks at $16\frac{3}{4}$ ct. a pound.

57. 580 lb. rabbits at $23\frac{7}{8}$ ct. a pound.

58. 240 lb. geese at $11\frac{1}{4}$ ct. a pound.

59. 370 lb. butter at $25\frac{1}{2}$ ct. a pound.

60. A wholesale dealer bought 1200 lb. of turkeys at $17\frac{1}{4}$ ct. a pound. He paid an average of $2\frac{1}{2}$ ct. a pound for freight. He sold the turkeys for 26 ct. a pound. Find his profit.

61. A dealer bought 960 bbl. of apples at $\$3\frac{1}{4}$ a barrel. He sold $\frac{1}{4}$ of them at $\$4\frac{1}{4}$ a barrel; $\frac{1}{2}$ of them at $\$4\frac{1}{2}$ a barrel and the remainder at $\$4\frac{3}{4}$ a barrel. Find the dealer's profits if his shipping expenses were $\$425$.

62. 320 bbl. of onions were bought at $\$2\frac{1}{4}$ a barrel. One half of them were sold at $\$3\frac{3}{4}$ a barrel; one half of the remainder were sold at $\$4\frac{1}{2}$ a barrel; the rest were sold at cost. Find the profit.

63. A grocer puts sugar up in packages, each package containing $3\frac{1}{2}$ lbs. If the grocer makes a profit of $2\frac{1}{2}$ ct. on each package, how much profit will he make on 644 lb. of sugar?

64. If $6\frac{1}{4}$ yd. of cloth cost $\$9\frac{1}{2}$, find the cost of $8\frac{1}{2}$ yd.

65. $3\frac{1}{4}$ strips of carpet, each strip containing $15\frac{1}{2}$ sq. yd., were used to carpet a room. Find the cost of the carpet at $\$1\frac{3}{4}$ a yard.

66. If a train travels $130\frac{1}{2}$ mi. in $6\frac{1}{4}$ hr., how far will it travel in $18\frac{3}{4}$ hr.?

67. $12\frac{1}{2}$ doz. eggs cost $\$3\frac{1}{2}$. Find the cost of 1 doz.

68. $18\frac{1}{2}$ lb. of butter cost $\$5\frac{1}{2}$. Find the cost of $55\frac{1}{2}$ lb.

69. $\frac{3}{8}$ of the distance between two cities is $150\frac{1}{8}$ mi. Find the entire distance.

70. A river is 640 mi. long. This is $\frac{2}{5}$ of the length of a second river. Find the length of the second river.

71. $\frac{4}{9}$ of the pupils in a school are boys. If there are 496 boys, how many pupils are in the school? How many girls?

72. The length of a rectangular plot of ground is 62 ft. The width is $2\frac{1}{4}$ times the length. Find the width. Find the area of the plot.

73. A man spent $\frac{3}{5}$ of his salary, or $\$27.60$, for clothing. Find his salary.

74. A tank contained 650 gal. of water. One half of the amount was drawn off; then $\frac{1}{4}$ of the remainder; then $\frac{1}{2}$ of what was left. How many gallons still remained in the tank?

75. One half of the contents of a tank were drawn off; then $\frac{1}{2}$ of the remainder; then $\frac{1}{3}$ of what was left. There were still 126 gal. in the tank. How many gallons were in the tank at first?

76. $\frac{5}{6}$ of A's salary equals \$86.50; $\frac{7}{8}$ of B's salary equals \$91. How much greater is B's salary than A's?

77. How many barrels of cement may be purchased for \$34 $\frac{1}{2}$, if a barrel of cement costs \$2 $\frac{7}{8}$?

78. How many pieces of rope each 1 $\frac{1}{2}$ ft. long may be cut from a piece 39 ft. long?

79. How long will it take a man to earn \$41 $\frac{1}{4}$ at the rate of \$2 $\frac{3}{4}$ per day?

80. How many bags of oats each containing 2 $\frac{1}{2}$ bu. can be filled from a bin that contains 115 bu.?

81. A vessel sailed $\frac{1}{4}$ of the distance between two ports the first day; $\frac{1}{5}$ of the distance, the second day; $\frac{1}{2}$ of the remainder, the third day. There were 75 mi. left. Find the total distance.

82. The difference between $\frac{7}{12}$ of a man's money and $\frac{3}{8}$ of his money is \$3655. Find the amount of his money.

83. A board is cut into 8 equal pieces. Then one of the pieces is cut into 4 equal parts. If the length of one of the smaller pieces is $\frac{3}{8}$ ft., what was the length of the original board?

84. A and B are 21 mi. apart, and are walking toward each other. A walks at the rate of $2\frac{1}{2}$ mi. an hour; B, at the rate of $2\frac{3}{4}$ mi. an hour. How long will it take them to meet each other?

85. (a) A can do a piece of work in 7 hr. What part of it can he do in 1 hr.?

(b) B can do a piece of work in $9\frac{1}{2}$ hr. What part of it can he do in 1 hr.?

86. A can do a piece of work in 5 hr.; B can do the same work in $7\frac{1}{2}$ hr. How long will it take both of them, working together, to do the work?

87. (a) A pipe fills a tank in 6 hr. What part of the tank will be filled in 1 hr.?

(b) A pipe empties a tank in 8 hr. What part of the tank will be emptied in 1 hr.?

88. A tank has two pipes. One will fill the tank in 5 hr., the other will empty it in 7 hr. How long will it take to fill the tank, if both pipes are opened at the same time?

89. The distance between A and B is $2\frac{1}{2}$ times the distance between C and D. If $\frac{2}{7}$ of the distance between C and D is $40\frac{1}{3}$ mi., what is the distance between A and B? (Draw a diagram.)

90. How many more miles in the distance between A and B than in the distance between C and D?

91. A train travels $20\frac{3}{4}$ mi. an hour. How many miles does it travel in $7\frac{3}{4}$ hr. at that rate?

92. A train travels $18\frac{3}{5}$ mi. an hour. In how many hours will it travel $120\frac{9}{10}$ mi.?

93. A train travels $112\frac{1}{8}$ mi. in $5\frac{3}{4}$ hr. How many miles does it travel in 1 hr.?

94. A train travels $46\frac{3}{4}$ mi. in $2\frac{1}{2}$ hr. How many miles does it travel in $7\frac{1}{2}$ hr.?

95. A train travels $58\frac{1}{2}$ mi. in $3\frac{1}{2}$ hr. How many miles does it travel in $5\frac{3}{4}$ hr.?

Compare 91, 92, 93, 94, 95, as to what is given, what is required, and what operation is performed.

96. $3\frac{1}{2}$ doz. handles cost \$1.80, how much will 18 doz. cost?

97. $20\frac{1}{2}$ rd. of wire cost \$3.70. Find the cost of 1 rd. of wire.

98. $16\frac{1}{2}$ rd. of wire weigh 78 lb. Find the weight of $5\frac{1}{2}$ rd.

PRICES OF BELTING

	PRICE PER FOOT	HOW MANY FEET FOR
99.	$1\frac{3}{4}$ ct	\$1.40
100.	$2\frac{3}{4}$	5.61
101.	$3\frac{3}{4}$	9.45
102.	$4\frac{1}{2}$	7.38

103. A tailor is paid $\$ \frac{3}{5}$ an hour. He works $9\frac{1}{2}$ hr. a day for 6 da. each week. How much does he receive for working $4\frac{1}{2}$ wk.?

104. The electric light used in a factory costs $\$ \frac{2}{5}$ every $3\frac{1}{2}$ hr. How much does it cost every 6 da., if light is used for $2\frac{1}{2}$ hr. a day?

105. The population of a city, 3 yr. ago, was 350,000. Each year, during the last three years, the population has increased $\frac{1}{10}$ of what it was the preceding year. Find the present population.

106. If an aëroplane flies 54 mi. in $2\frac{3}{4}$ hr., how many miles will it fly in $5\frac{1}{2}$ hr.?

107. If an aëroplane flies 49 mi. in $2\frac{1}{3}$ hr., in how many hours will it fly 70 mi.?

108. A merchant has two qualities of cloth. The better quality costs $2\frac{1}{2}$ times as much as the poorer quality. If $\frac{3}{4}$ yd. of the poorer quality costs $\$ \frac{3}{4}$, find the cost of $5\frac{1}{2}$ yd. of the better quality.

109. $\frac{3}{4}$ lb. of coffee costs $\$ \frac{3}{8}$.

(a) Find the cost of $\frac{1}{2}$ lb.

(b) Find the cost of $4\frac{1}{2}$ lb.

(c) Find how many pounds may be bought for $\$ 2\frac{1}{2}$.

110. $\frac{1}{2}$ yd. of ribbon cost 40 ct.

(a) What part of a yard may be bought for 20 ct.?

(b) How many yards may be bought for $\$ 2$?

(c) Find the cost of $5\frac{1}{2}$ yd.

111. I bought 120 bbl. of flour at \$4 per barrel. The price is now \$6 per barrel. How many barrels can I buy now for the same amount of money?

112. How long will it take a man earning \$4 $\frac{1}{2}$ a day to save \$180, if he spends $\frac{3}{5}$ of his wages?

113. A gardener and his son hoe 2800 plants in 3 $\frac{1}{2}$ da., the son hoeing $\frac{3}{5}$ as many as his father. What was the average number of plants per day hoed by the father? By the son?

114. A farm of 2,400 A. has 660 A. under cultivation, 720 A. in pasture, and 980 A. in timber; the rest is fallow. What part of the farm is fallow?

115. A hardware merchant bought 360 pairs of steel skates at \$1 $\frac{1}{8}$, and 480 pairs of roller skates at \$ $\frac{7}{8}$ per pair; he sold the steel skates at \$1 $\frac{3}{4}$ per pair, and the roller skates at \$1 $\frac{1}{4}$ per pair. Find his total profit.

116. How much should a dealer receive for 100 gross of lamp chimneys at \$ $\frac{5}{8}$ per dozen?

117. How many books bought at the rate of 6 for \$1 must be sold at the rate of 4 for a dollar in order to realize a profit of \$60?

118. What should a farmer get for 1750 bu. of onions, if he sells them at the rate of \$2 $\frac{3}{4}$ per box of 1 $\frac{3}{4}$ bu.?

119. An orchard yields 1570 bbl. of apples, of which $\frac{3}{5}$ will grade as "eating apples," $\frac{3}{10}$ as "cooking apples," and the remainder as "cider apples." How many barrels of each grade are there in the crop?

120. In 3 da. a commissary officer supplied 270 lb. of beef to a company of 120 men. How much beef was allowed for each man per day?

121. A cubic foot of water weighs $62\frac{1}{2}$ lb. A tank has a capacity of $21\frac{3}{4}$ cu. ft. What is the weight of water which it will contain?

122. A contractor has done $\frac{3}{5}$ of a job in $12\frac{3}{4}$ da.; how much longer should it take to finish the job?

123. $\frac{1}{3}$ of $\frac{2}{5}$ of a certain number equals 20; what is the number?

124. Two ships sail from Honolulu at the same time; one sails in a westerly direction at the rate of $15\frac{3}{8}$ mi. an hour; the other sails in an easterly direction at the rate of $18\frac{2}{3}$ mi. an hour. How far apart will the ships be after traveling $5\frac{1}{3}$ hr.?

125. The distance from New York to Albany is $\frac{3}{20}$ of the distance from New York to Chicago. Assuming that Albany is 144 mi. from New York, ascertain the distance from New York to Chicago.

126. A train travels $87\frac{1}{2}$ mi. less in $4\frac{3}{4}$ hr. than it does in $6\frac{1}{3}$ hr.; what is its rate of speed?

127. $\frac{3}{7}$ of John's money is equal to $\frac{5}{16}$ of Jane's. What part of Jane's money is John's equal to?

128. A farmer exchanged $6\frac{3}{4}$ lb. of butter for $4\frac{1}{2}$ lb. of tea. If the tea was worth 60 ct. a pound, what value per pound did the farmer receive for the butter?

129. A grocer sold $\frac{3}{8}$ of a cask of vinegar; $\frac{1}{8}$ of what remained measured $2\frac{3}{4}$ gal. How many gallons in the full cask?

130. A man purchased 10 T. of white ash coal at $\$5\frac{3}{4}$ a ton, and half as many tons again of red ash coal at $\$ \frac{1}{2}$ more per ton. What was the amount of his bill?

131. It required 1150 ft. of fencing to inclose a rectangular plot of ground; $\frac{1}{8}$ of the width of the plot equals $\frac{1}{15}$ of its length. What are the dimensions of the plot?

132. In a 5-A. field there is a plot 12 rd. by 15 rd. planted in vegetables. What part of the field is the vegetable garden?

133. An automobile consumes $18\frac{3}{4}$ gal. of gasoline in going 75 mi. Find the average consumption per mile, and the cost per mile if gasoline is worth 20 ct. a gallon?

DECIMAL FRACTIONS

42. A decimal fraction is a fraction whose denominator is ten or a power of ten; *e.g.* 10, 100, 1000, etc.

The denominator of a decimal fraction is rarely expressed in figures. It is understood and read by its position in relation to the decimal point.

Ten orders of whole numbers and eight orders of decimals are shown in the following table:

4 Billions	0 Hundred-millions	7 Ten-millions	0 Millions	0 Hundred-thousands	0 Ten-thousands	0 Thousands	0 Hundreds	4 Tens	0 Units	•	0 Tenths	7 Hundredths	0 Thousandths	0 Ten-thousandths	0 Hundred-thousandths	0 Millionths	4 Ten-Millionths	0 Hundred-millionths	
ORDERS OF WHOLE NUMBERS											ORDERS OF DECIMALS								

The denominator of a decimal fraction is the name of the order to the right of the decimal point occupied by the last figure of the numerator.

PRINCIPLES. 1. The value of the orders of decimals *decreases* from left to right and *increases* from right to left in the same manner as the orders of whole numbers.

2. The removal of a decimal figure one place to the *left*, *multiplies* its value by 10, and its removal one place to the *right*, *divides* its value by ten; *e.g.* .40 multiplied by 10 becomes 4.0; and 0.5 divided by 10 becomes .05.

NOTE. Moving the decimal point to the right produces the same result as moving the figure to the left; moving the decimal point to the left produces the same result as moving the figure to the right.

FORMS OF REDUCTION USED IN DECIMALS

43. 1. To change a decimal to a lower decimal order.

$$.5 = .50 \text{ or } .500 \quad .07 = .070 \text{ or } .07000$$

2. To change a decimal to a higher decimal order.

$$.750 = .75 \quad .06400 = .064 \quad .7000 = .7$$

3. To change a decimal to an equivalent common fraction.

$$.25 = \frac{25}{100} = \frac{1}{4} \quad .66\frac{2}{3} = \frac{200}{300} = \frac{2}{3} \quad .075 = \frac{75}{1000}$$

4. To change a common fraction to an equivalent decimal.

$$\frac{7}{8} = .87\frac{1}{2} \text{ or } .875 \quad \frac{1}{3} = .33\frac{1}{3} \quad \frac{1}{200} = .005$$

ORAL EXERCISES

44 1. How many .00's in $\frac{1}{2}$? In $\frac{1}{6}$? In $\frac{1}{10}$? In $\frac{1}{4}$? In $\frac{1}{20}$?

2. Change to common fractions in their lowest or smallest terms:

$$.40; .75; .30; .25; .90$$

3. Change to highest decimal orders:
 .600; .250; .3170; .0800; .0400
4. Change to .0000's:
 .5; .17; .025; .21; .90

WRITTEN EXERCISES

1. Change to fractions of three decimal orders:
 $\frac{1}{3}, \frac{1}{7}, \frac{1}{9}, \frac{1}{12}, \frac{1}{8}$.
2. Change to .000's:
 $\frac{2}{7}, \frac{3}{8}, \frac{5}{9}, \frac{7}{12}, \frac{3}{16}, \frac{8}{15}, \frac{3}{11}, \frac{5}{6}, \frac{7}{9}, \frac{11}{16}$.
3. Change to common fractions in their lowest terms:
 .375; .3125; .625; .1875; .0875
4. Change to common fractions in their lowest terms:
 $.06\frac{2}{3}; .833\frac{1}{3}; .0112\frac{1}{2}; .07\frac{1}{2}; .014\frac{2}{7}$

EXERCISE

DRILL IN SHIFTING DECIMAL POINTS

1. Read 27.135.
2. Place the decimal point between 1 and 3.
 Now read the number.
3. Between 2 and 7. Read the new number.
4. Between 3 and 5. Read the new number.
5. Place the decimal point before 2. Read.
6. Place the decimal point after 5. Read.

7. Change the position of the point in 370.993 six different times and read the new number each time.

8. Read 700.07. Change the position of the point four different times to the left, and four different times to the right, and read the new number each time.

45. Memorize the following fractional parts of a dollar.

5 cents	= $\frac{1}{20}$ of a dollar	= \$.05
$8\frac{1}{3}$ cents	= $\frac{1}{12}$ of a dollar	= .08 $\frac{1}{3}$
10 cents	= $\frac{1}{10}$ of a dollar	= .10
$12\frac{1}{2}$ cents	= $\frac{1}{8}$ of a dollar	= .12 $\frac{1}{2}$
$16\frac{2}{3}$ cents	= $\frac{1}{6}$ of a dollar	= .16 $\frac{2}{3}$
20 cents	= $\frac{1}{5}$ of a dollar	= .20
25 cents	= $\frac{1}{4}$ of a dollar	= .25
30 cents	= $\frac{3}{10}$ of a dollar	= .30
$33\frac{1}{3}$ cents	= $\frac{1}{3}$ of a dollar	= .33 $\frac{1}{3}$
$37\frac{1}{2}$ cents	= $\frac{3}{8}$ of a dollar	= .37 $\frac{1}{2}$
40 cents	= $\frac{2}{5}$ of a dollar	= .40
50 cents	= $\frac{1}{2}$ of a dollar	= .50
60 cents	= $\frac{3}{5}$ of a dollar	= .60
$62\frac{1}{2}$ cents	= $\frac{5}{8}$ of a dollar	= .62 $\frac{1}{2}$
$66\frac{2}{3}$ cents	= $\frac{2}{3}$ of a dollar	= .66 $\frac{2}{3}$
70 cents	= $\frac{7}{10}$ of a dollar	= .70
75 cents	= $\frac{3}{4}$ of a dollar	= .75
80 cents	= $\frac{4}{5}$ of a dollar	= .80
$87\frac{1}{2}$ cents	= $\frac{7}{8}$ of a dollar	= .87 $\frac{1}{2}$

ADDITION OF DECIMALS

EXERCISES

46. Add at sight:

- | | |
|--------------------|------------------------------|
| 1. .2, .07, .5. | 4. 4.6 and $2\frac{1}{2}$. |
| 2. 2.6, 3.1, 4.2. | 5. 50.4 and 10.07. |
| 3. 2.5, 3.7, 1.05. | 6. 2.65 and $3\frac{1}{4}$. |

WRITTEN EXERCISES

Add:

1.	2.	3.
175.046	2000.002	17.0942
22.8	70.7	864.18
2095.0743	496.2938	29.046
876.21	27.046	$793\frac{1}{2}$
5.099	299.0011	$86.09\frac{1}{2}$
613.2	$17\frac{1}{2}$	$2.064\frac{1}{2}$
<hr/>	<hr/>	<hr/>

4.

5.

6.

Make up three examples of 8 addends each, the greatest denominators being 100, 1000, and 10000, respectively.

SUBTRACTION OF DECIMALS

ORAL EXERCISES

47. 1. Subtract at sight, 3.15 from 7.29.
 2. Subtract at sight, 18.07 from 26.13.
 Prove examples 1 and 2.

3. Subtract at sight, 13.48 from 32.
4. Subtract at sight, 7.019 from 25.16.
5. Subtract at sight, 42.008 from 90.
6. Subtract at sight, 31.117 from 74.006.

WRITTEN EXERCISES

1.	2.	3.
From $100\frac{1}{2}$	From 175.18	From 1000.001
take <u>72.086</u>	take <u>$76\frac{1}{8}$</u>	take <u>93.0089</u>

4.	5.	6.
From $1000\frac{1}{8}$	From 1111.01	From 1000
take <u>273.18</u>	take <u>783.968</u>	take <u>.001</u>

7. Find the difference between 1762.0018 and $963\frac{7}{8}$.
8. Find the difference between 1.009 and $1.0\frac{1}{2}$.
9. Find the difference between .75 and 75.
10. Find the difference between 2000 and 1674.823.

MULTIPLICATION OF DECIMALS

DIFFERENT PROCESSES USED IN MULTIPLICATION OF DECIMALS

- 48.** 1. To multiply a decimal by a whole number.
Multiply: .6 by 12; .008 by 32; .164 by 11.

$$\begin{array}{r} .6 \\ 12 \\ \hline 7.2 \end{array}$$

$$\begin{array}{r} .008 \\ 32 \\ \hline .256 \end{array}$$

$$\begin{array}{r} .164 \\ 11 \\ \hline 1.804 \end{array}$$

Multiply: 7.4×26 ; 18.02×32 ; 17.009×14 .

$$\begin{array}{r} 7.4 \\ 26 \\ \hline 444 \\ 148 \\ \hline 192.4 \end{array}$$

$$\begin{array}{r} 18.02 \\ 32 \\ \hline 3604 \\ 5406 \\ \hline 576.64 \end{array}$$

$$\begin{array}{r} 17.009 \\ 14 \\ \hline 68036 \\ 17009 \\ \hline 238.126 \end{array}$$

2. To multiply a decimal by a whole number ending in one or more ciphers.

A. Multiply 3.75 by 10; by 100; by 1000.

$$\begin{array}{r} 3.75 \\ 10 \\ \hline 37.50 \end{array}$$

$$\begin{array}{r} 3.75 \\ 100 \\ \hline 375.00 \end{array}$$

$$\begin{array}{r} 3.75 \\ 1000 \\ \hline 3750.00 \end{array}$$

PRINCIPLE. The removal of the decimal point *one place* to the *right*, *multiplies* the decimal by *10*; the removal of the decimal point *two places* to the *right*, *multiplies* the decimal by 10 times 10, or *100*; the removal of the decimal point *three places* to the *right*, *multiplies* the decimal by 10 times 100, or *1000*; and so on.

Multiply: 6.4×20 ; $.263 \times 300$; 7.025×750 .

$$\begin{array}{r} 6 \times 4. \\ 20 \\ \hline 128. \end{array}$$

$$\begin{array}{r} \times 26.3 \\ 300 \\ \hline 78.9 \end{array}$$

$$\begin{array}{r} 7 \times 0.25 \\ 750 \\ \hline 35125 \\ 49175 \\ \hline 5268.75 \end{array}$$

Note that in each example, the decimal point is moved as many places to the right as there are zeros at the right of the multiplier.

3. To multiply a whole number by a decimal.

Find: .7 of 63; .18 of 25; .009 of 156.

$$\begin{array}{r} 63 \\ .7 \\ \hline 44.1 \end{array}$$

$$\begin{array}{r} 25 \\ .18 \\ \hline 200 \\ 25 \\ \hline 4.50 \end{array}$$

$$\begin{array}{r} 156 \\ .009 \\ \hline 1.404 \end{array}$$

Multiply: 726 by 8.2; 64 by 1.005; 826 by 1.3.

$$\begin{array}{r} 726 \\ 8.2 \\ \hline 1452 \\ 5808 \\ \hline 5953.2 \end{array}$$

$$\begin{array}{r} 64 \\ 1.005 \\ \hline 320 \\ 64 \\ \hline 64.320 \end{array}$$

$$\begin{array}{r} 826 \\ 1.3 \\ \hline 2478 \\ 826 \\ \hline 1073.8 \end{array}$$

4. To multiply a decimal by a decimal.

Find: .7 of .125; .008 of .25; .08 of 2.7.

$$\begin{array}{r} .125 \\ .7 \\ \hline .0875 \end{array}$$

$$\begin{array}{r} .25 \\ .008 \\ \hline .00200 \end{array}$$

$$\begin{array}{r} 2.7 \\ .08 \\ \hline .216 \end{array}$$

Multiply: 26.4 by .005; 84.4 \times 9.05.

$$\begin{array}{r} 26.4 \\ .005 \\ \hline .1320 \end{array}$$

$$\begin{array}{r} 84.4 \\ 9.05 \\ \hline 4220 \\ 7596 \\ \hline 763.820 \end{array}$$

49. RULE. Multiply as with whole numbers, and point off as many places from the right of the product as the sum of the decimal places in both multiplicand and multiplier.

ORAL EXERCISES

- | | |
|-------------------------|---------------------------|
| 50. 1. Multiply 9 by 5 | 2. Multiply .12 by 6 |
| 3. Multiply .15 by 7 | 4. Multiply .08 by 4 |
| 5. Multiply .25 by 6 | 6. Multiply .02 by 12 |
| 7. Multiply .30 by 10 | 8. Multiply .49 by 2 |
| 9. Multiply .78 by 3 | 10. Multiply .85 by 10 |
| 11. Multiply 16 by .2 | 12. Multiply 25 by .3 |
| 13. Multiply 7 by .04 | 14. Multiply 9 by .06 |
| 15. Multiply 12 by .12 | 16. Multiply 10 by .14 |
| 17. Multiply 28 by .03 | 18. Multiply 45 by .06 |
| 19. Multiply 70 by .07 | 20. Multiply 55 by .008 |
| 21. Multiply .2 by .5 | 22. Multiply .4 by .8. |
| 23. Multiply .15 by .6 | 24. Multiply .18 by .06 |
| 25. Multiply .22 by .05 | 26. Multiply 2.5 by 8 |
| 27. Multiply 4.6 by 3 | 28. Multiply 15.2 by 5 |
| 29. Multiply 9 07 by 7 | 30. Multiply 12.3 by 9 |
| 31. Multiply 3.85 by 10 | 32. Multiply 3.85 by 100 |
| 33. Multiply 4.9 by 20 | 34. Multiply 4.09 by 200 |
| 35. Multiply 2.10 by 30 | 36. Multiply 12 by 1.6 |
| 37. Multiply 7 by 2.5 | 38. Multiply 5 by 3.6 |
| 39. Multiply 8 by 4.3 | 40. Multiply 11 by 6.5 |
| 41. Multiply 1.5 by 1.5 | 42. Multiply 2.6 by .04 |
| 43. Multiply 2.7 by 1.1 | 44. Multiply 3.01 by .3 |
| 45. Multiply 8.01 by .1 | 46. Multiply 15.13 by .01 |
| 47. Multiply 9.2 by 1.2 | 48. Multiply 4.01 by 1.2 |

WRITTEN EXERCISES

1. Multiply .275 by 21
2. Multiply .508 by 18
3. Multiply .418 by 37
4. Multiply .3007 by 85
5. Multiply .649 by 28
6. Multiply .5376 by $73\frac{3}{4}$
7. Multiply .461 by 79
8. Multiply .3105 by $27\frac{1}{3}$
9. Multiply .7006 by 39
10. Multiply .2514 by $34\frac{1}{2}$
11. Multiply 135 by .142
12. Multiply 295 by .106
13. Multiply 732 by .1305
14. Multiply 829 by .368
15. Multiply 694 by .208
16. Multiply 365 by .129
17. Multiply 582 by .4009
18. Multiply 135 by .165
19. Multiply 238 by .1029
20. Multiply 2005 by .1008

Find the products :

21. $.295 \times .16$

22. $.147 \times .27$

23. $.631 \times .47$

24. $.278 \times .109$

- | | |
|--|------------------------------------|
| 25. $.306 \times .105$ | 26. $27.4 \times .009$ |
| 27. 11.5×37 | 28. 16.02×29 |
| 29. 13.08×42 | 30. 20.06×20 |
| 31. 31.08×4000 | 32. 3.004×1050 |
| 33. 40.09×200 | 34. 100.6×500 |
| 35. 305.01×40 | 36. 245×1.15 |
| 37. 126×2.43 | 38. 219×3.18 |
| 39. 491×3.78 | 40. 502×14.06 |
| 41. 298×24.01 | 42. 12.09×3.95 |
| 43. 42.19×6.31 | 44. 63.05×19.63 |
| 45. 40.04×6.03 | 46. 55.27×2.79 |
| 47. 44.07×20.09 | 48. 21.95×2.05 |
| 49. 29.16×3.84 | 50. 38.41×30.04 |
| 51. $723.8 \times 72\frac{1}{3}$ | 52. $.625 \times 6.25$ |
| 53. $6.486 \times 400\frac{1}{6}$ | 54. $.072 \times 10.1$ |
| 55. $7200 \times .047$ | 56. 864×1.009 |
| 57. 372.8×4000 | 58. $92.17 \times .005$ |
| 59. $68\frac{3}{4} \times 9.72\frac{1}{2}$ | 60. 43.85×246.7 |
| 61. 3.097×800 | 62. 6.29×18.004 |
| 63. $.42\frac{1}{3} \times 7000$ | 64. $76.48 \times 1.01\frac{1}{3}$ |
| 65. 36.8×2950 | 66. 628.43×70 |
| 67. 35.008×17.6 | 68. $6.29\frac{1}{3} \times 100.2$ |
| 69. $40.872 \times .015$ | 70. 1800×1.0007 |
| 71. $176.29 \times .003$ | 72. 296.13×1.0302 |
| 73. 58.021×7.09 | 74. 594.19×4.090 |

DIVISION OF DECIMALS

Division by a Whole Number

ORAL EXERCISES

51. 1. Divide by 2: 6; 6 tenths; .6; 6 hundredths; .06; 006.

2. Divide by 4: .24; .024; .28; .028; 2.4; 2.8.

3. Divide by 5: .5 or 50; .6 or 60; .07 or .070.

4. Divide by 6: 72 yd.; 120 ct.; 156 thousandths; 1266 hundredths.

5. Find quotients:

$$6 \overline{) 72 \text{ yd.}}$$

$$6 \overline{) \$1.20}$$

$$6 \overline{) \$7.92}$$

$$6 \overline{) \$27.30}$$

ORAL EXERCISES

52. Find quotients quickly:

1. $4.8 \div 4$

5. $.075 \div 15$

9. $.54 \div 6$

2. $.036 \div 6$

6. $63.7 \div 7$

10. $7.2 \div 8$

3. $.64 \div 8$

7. $.81 \div 9$

11. $.0018 \div 9$

4. $1.44 \div 12$

8. $1.25 \div 5$

12. $.096 \div 12$

Division by 10, 100, 1000, etc.

53. Using short division, divide .36 by 10; 27.4 by 100; 723.8 by 1000.

$$\begin{array}{r} 10 \overline{) .360} \\ .036 \end{array}$$

or

$$.36 \div 10 = .036$$

PROOF

$$.036 \times 10 = .360$$

$$\begin{array}{r} 100 \overline{) 27.400} \\ .274 \end{array}$$

or

$$27.4 \div 100 = .274$$

PROOF

$$.274 \times 100 = 27.400$$

$$\begin{array}{r} 1000 \overline{) 723.8000} \\ .7238 \end{array}$$

or

$$723.8 \div 1000 = .7238$$

PROOF

$$.7238 \times 1000 = 723.8000$$

It has already been shown (page 69) that to *multiply* a decimal by 10, or 100, or 1000, the decimal point is moved one, or two, or three places to the *right*. Division is the reverse of multiplication.

RULE. To *divide* a number by 10, or 100, or 1000, the decimal point is moved *one*, or *two*, or *three* places to the *left*.

Decimal ciphers are prefixed, when necessary, to give the quotient the proper number of decimal places.

ORAL EXERCISES

Divide by 10:

1. 76.8
2. 276.42
3. 75.18
4. 300.9
5. 7258.11

Divide by 100:

6. 6428.5
7. 720.91
8. 86.487
9. 290.83
10. 1194.2

Divide by 1000:

- | | |
|-------------|-------------|
| 11. 26849.3 | 14. 298.462 |
| 12. 829.46 | 15. 1799.34 |
| 13. 700.863 | 16. 82927.2 |

54.

WRITTEN EXERCISES

- | | |
|------------------------|--------------------------|
| 1. $76.45 \div 30 = ?$ | 5. $57.68 \div 80 = ?$ |
| 2. $389.2 \div 70 = ?$ | 6. $86.433 \div 300 = ?$ |
| 3. $6.495 \div 50 = ?$ | 7. $176.75 \div 500 = ?$ |
| 4. $728.4 \div 40 = ?$ | 8. $4956.7 \div 700 = ?$ |

Division by a Decimal

55. Divide:

$$1. \ 4 \overline{)8} \quad 2. \ .4 \overline{).8} \quad 3. \ .04 \overline{).08} \quad 4. \ .004 \overline{).008}$$

$$5. \text{ Explain } \left\{ \begin{array}{l} .8 \div .2 = \\ .08 \div .02 = \\ .008 \div .002 = \end{array} \right. \rightarrow 8 \div 2 = 4.$$

If the divisor is multiplied by 10, what must be done to the dividend to secure the same quotient? If the divisor is multiplied by 100? By 1000?

6. How may $.8 \div .2$ be changed to $8 \div 2$?7. How may $.08 \div .02$ be changed to $8 \div 2$?8. How may $.008 \div .002$ be changed to $8 \div 2$?

This recalls the important

PRINCIPLE. Multiplying both dividend and divisor by the same number does not change the quotient.

SUMMARY OF DIVISION OF DECIMALS

DIFFERENT PROCESSES USED IN DIVISION OF DECIMALS

SHORT DIVISION

56. 1. To divide a decimal by a whole number.

$$.375 \div 5$$

$$\begin{array}{r} 5 \overline{).375} \\ \underline{*075} \end{array}$$

$$26.084 \div 4$$

$$\begin{array}{r} 4 \overline{)26.084} \\ \underline{6*521} \end{array}$$

* Always place the decimal point in the quotient *before* beginning to divide.

LONG DIVISION

$.0875 \div 25$

$$\begin{array}{r}
 *0035 \\
 25 \overline{) .0875} \\
 \underline{75} \\
 125 \\
 \underline{125} \\
 0000
 \end{array}$$

$164.018 \div 32$

$$\begin{array}{r}
 5*125 \frac{9}{16} \\
 32 \overline{) 164.018} \\
 \underline{160} \\
 40 \\
 \underline{32} \\
 81 \\
 \underline{64} \\
 178 \\
 \underline{160} \\
 18 \\
 32 \overline{) 18} = 1 \frac{1}{2}
 \end{array}$$

2. To divide a decimal by 10, 100, 1000.

$.42 \div 10; 15.6 \div 100; 612.5 \div 1000.$

$ \begin{array}{r} 10 \overline{) .420} \\ \underline{.042} \\ 000 \end{array} $	$ \begin{array}{r} 100 \overline{) 15.600} \\ \underline{.156} \\ 000 \end{array} $	$ \begin{array}{r} 1000 \overline{) 612.5000} \\ \underline{.6127} \\ 0000 \end{array} $
or	or	or
$.42 \div 10 = .042$	$15.6 \div 100 = .156$	$612.5 \div 1000 = .6125$

RULE. To *divide* a number by 10, 100, or 1000, the decimal point is moved *one*, or *two*, or *three places* to the *left*.

Decimal ciphers are prefixed to the quotient, when necessary, to give the proper number of decimal places.

3. To divide a whole number by a decimal.

$360 \div .18$

$$\begin{array}{r}
 2000* \\
 \times 18 \overline{) 360 \times 00} \\
 \underline{36} \\
 0000
 \end{array}$$

$264 \div 4.4$

$$\begin{array}{r}
 60* \\
 4 \times 4 \overline{) 264 \times 0} \\
 \underline{264} \\
 000
 \end{array}$$

* Always place the decimal point in the quotient *before* beginning to divide.

4. To divide a decimal by a decimal.

$$.125 \div .5$$

$$\begin{array}{r} \times 5. \overline{) \times 1.25} \\ \underline{\times 25} \end{array}$$

$$.1728 \div 2.4$$

$$\begin{array}{r} \times 072 \\ 2 \times 4. \overline{) \times 1.728} \\ \underline{168} \\ 48 \\ \underline{48} \end{array}$$

$$345.6 \div .012$$

$$\begin{array}{r} \times 012. \overline{) 345.600} \\ \underline{28800}^* \end{array}$$

$$34.56 \div 7.2$$

$$\begin{array}{r} \times 4.8 \\ 7 \times 2. \overline{) 34.56} \\ \underline{288} \\ 576 \\ \underline{576} \end{array}$$

5. To divide a decimal or a whole number by a complex decimal.

$$.125 \div .3\frac{2}{3}$$

$$\begin{array}{r} \times 3\frac{2}{3}. \overline{) \times 1.25} \\ \underline{3} \quad \underline{3} \\ 11 \overline{) 3.75} \\ \underline{34} \frac{1}{11} \end{array}$$

$$2000 \div .99\frac{1}{3}$$

$$\begin{array}{r} \times 99\frac{1}{3}. \overline{) 2000 \times 00} \\ \underline{3} \quad \underline{3} \\ 298 \overline{) 600000} \end{array}$$

divide as in whole numbers.

$$27.14 \div .02\frac{1}{2}$$

$$\begin{array}{r} \times 02\frac{1}{2}. \overline{) 27 \times 14} \\ \underline{7} \quad \underline{7} \\ 15 \overline{) 18998.00} \\ \underline{1266.53} \frac{1}{8} \end{array}$$

$$7500 \div 1.02\frac{5}{6}$$

$$\begin{array}{r} 1 \times 02\frac{5}{6}. \overline{) 7500 \times 00} \\ \underline{6} \quad \underline{6} \\ 617 \overline{) 4500000} \end{array}$$

divide as in whole numbers.

57. RULE. 1. Change the divisor to a whole number by moving the decimal point to the right of the last figure.

2. Move the decimal point in the dividend as many places to the right as the decimal point was moved in the divisor ; if necessary, annex ciphers to the dividend.

* Always place the decimal point in the quotient *before* beginning to divide.

3. Place the decimal point in the quotient directly above the decimal point in the dividend.

4. Divide as with whole numbers.

ORAL EXERCISES

- | | |
|-----------------------|-----------------------|
| 58. 1. Divide 15 by 3 | 2. Divide .15 by 5 |
| 3. Divide .21 by 7 | 4. Divide .32 by 4 |
| 5. Divide .08 by 4 | 6. Divide .36 by 9 |
| 7. Divide .035 by 5 | 8. Divide .045 by 9 |
| 9. Divide .213 by 3 | 10. Divide .168 by 8 |
| 11. Divide 240 by .3 | 12. Divide 240 by 4 |
| 13. Divide 150 by .5 | 14. Divide 150 by .15 |
| 15. Divide 279 by .9 | 16. Divide 336 by .8 |
| 17. Divide 252 by .06 | 18. Divide 165 by .05 |
| 19. Divide 252 by 1.2 | 20. Divide 330 by 1.1 |

Find quotients :

- | | |
|--------------------|---------------------|
| 21. $.34 \div .2$ | 22. $.80 \div .5$ |
| 23. $.35 \div 7$ | 24. $.24 \div .3$ |
| 25. $.06 \div .2$ | 26. $.21 \div .3$ |
| 27. $1.2 \div .4$ | 28. $1.2 \div .6$ |
| 29. $1.8 \div .9$ | 30. $3.6 \div .9$ |
| 31. $5.4 \div .9$ | 32. $1.11 \div .3$ |
| 33. $4.2 \div 10$ | 34. $4.2 \div .1$ |
| 35. $4.2 \div .01$ | 36. $5.6 \div 2$ |
| 37. $5.6 \div .2$ | 38. $5.6 \div .02$ |
| 39. $47.3 \div 10$ | 40. $47.3 \div 100$ |

41. $47.3 \div 1000$

42. $5.64 \div 20$

43. $5.64 \div 200$

44. $5.64 \div 2000$

45. $7.5 \div 1.5$

46. $10.8 \div 1.2$

47. $10.8 \div 1.8$

48. $7.8 \div 1.3$

49. $8.5 \div 1.7$

50. $1.14 \div 1.9$

WRITTEN EXERCISES

1. Divide .576 by 16
2. Divide .1204 by 28
3. Divide .578 by 17
4. Divide .714 by 34
5. Divide .987 by 21
6. Divide .728 by 56
7. Divide .2546 by 38
8. Divide .3528 by 49
9. Divide 296 by .8
10. Divide 696 by .24
11. Divide .216 by .6 ; by .06 ; by .006.
12. Divide 2.835 by .07 ; by 7 ; by .007.
13. Divide 556 by .21.
14. Divide 814 by .37.
15. Divide 957 by 3.3.
16. Divide 1148 by 2.8
17. Divide .6732 by .18 ; by 1.8 ; by .018.
18. Divide .03125 by 12.5 ; by .125 ; by .0125.
19. Divide .0814 by .022 ; by 2.2 ; by .0022.
20. Divide 27.82 by 1.07 ; by .0107 ; by .107.
21. $.117 \div .13 = ?$
22. $.112 \div .16 = ?$
23. $.133 \div .19 = ?$
24. $.184 \div .23 = ?$
25. $.828 \div .16 = ?$
26. $.1608 \div .67 = ?$
27. $.1924 \div .26 = ?$
28. $.2856 \div .84 = ?$
29. $.2183 \div .59 = ?$
30. $.4368 \div .91 = ?$

Find quotients:

- | | |
|-----------------------------------|----------------------------------|
| 31. $.432 \div 1.6$ | 32. $.425 \div 1.7$ |
| 33. $2.322 \div 2.7$ | 34. $1.404 \div 3.6$ |
| 35. $1.645 \div 3.5$ | 36. $7.904 \div 1.04$ |
| 37. $1.05 \div 3.75$ | 38. $6.194 \div 3.8$ |
| 39. $2.9087 \div 2.9$ | 40. $23.435 \div 1.09$ |
| 41. $9.844 \div 1.02$ | 42. $9.399 \div 2.41$ |
| 43. $.015704 \div .013$ | 44. $.003213 \div .009$ |
| 45. $.001524 \div .012$ | 46. $10.165 \div .008$ |
| 47. $4.38 \div .012$ | 48. $210.672 \div 3.08$ |
| 49. $6.1194 \div 1.302$ | 50. $220.671 \div .107$ |
| 51. $1000 \div 3\frac{1}{3}$ | 52. $750.42 \div .00\frac{5}{8}$ |
| 53. $720.01 \div 1.02\frac{1}{2}$ | 54. $7000 \div .98\frac{1}{3}$ |
| 55. $204.75 \div 1.01\frac{5}{6}$ | 56. $1750 \div 1.18\frac{3}{4}$ |
| 57. $601.83 \div 1.00\frac{1}{3}$ | 58. $350 \div .007$ |
| 59. $4005 \div 1.007$ | 60. $80 \div .0004$ |

Perform the operations indicated by the signs

61. $26 + 18.9 \times 1.025$
62. $385.4 - 27.18 \div 2.5$
63. $7000 + 86.75 \div .015$
64. $7.2 \times 7.2 \div .0012$
65. $394.6 - 70.375 + 18.46 \times 20.7$
66. $13.4 + 78.315 \times 4.8 \div 200$
67. $77.4 \times 3.7\frac{1}{3} \div 100$
68. $2000 \div .99\frac{1}{3}$

TYPE PROBLEMS INVOLVING FRACTIONAL RELATIONS OF NUMBERS. (Decimal Fractions)

There are three types of problems involving multiplication and division of common fractions. (See pages 43 and 44.) The same three types of problems are found in multiplication and division of decimal fractions.

First Type Problem : To find a part of a number

59. COMMON FRACTIONS. A carpenter receives \$24 for a week's work. How much does he receive for working $\frac{5}{8}$ of a week?

DECIMAL FRACTIONS. A laborer mixed 250 lb. of mortar. .84 of it was sand. How many pounds of sand were used?

Explanations

COMMON FRACTIONS
The amount received for $\frac{5}{8}$ of a week is $\frac{5}{8}$ of the amount received for a week. Therefore the amount is
 $\frac{5}{8}$ of \$24 = \$20 *Ans.*

DECIMAL FRACTIONS
The amount of sand is .84 of the amount of mortar. Therefore the amount of sand is .84 of 250 lb., or
.84 \times 250 lb. = 210 lb. *Ans.*

Second Type Problem : To find what fraction one number is of another number

60. COMMON FRACTIONS. A storekeeper had 70 packages of oatmeal; he sold 35 packages. What part of his stock did he sell?

DECIMAL FRACTIONS. A storekeeper had 74 packages of oatmeal; he sold 35 packages. What part of his stock (expressed in decimals) did he sell?

Explanations

COMMON FRACTIONS

The part sold is 35 packages out of 70 packages, or $\frac{35}{70}$ or $\frac{1}{2}$ of the total stock.

DECIMAL FRACTIONS

The part sold is 35 packages out of 74 packages, or $\frac{35}{74}$ or $.47\frac{1}{4}$ of the total stock.

Third Type Problem: To find a number when a fractional part of it is given

61. COMMON FRACTIONS. $\frac{3}{5}$ of the number of pupils in a class is 27. How many pupils are there in the class?

DECIMAL FRACTIONS. .76 of the population of a town is 4066. What is the population?

Explanations

COMMON FRACTIONS

$\frac{3}{5}$ of the number of pupils is 27. Therefore $\frac{1}{5}$ of the number of pupils equals $\frac{1}{3}$ of 27.

All the pupils will equal 5 times ($\frac{1}{3}$ of 27) or

$$\frac{5}{3} \text{ of } 27 = 45 \text{ Ans.}$$

A shorter explanation is: $\frac{3}{5}$ of the number of pupils equals 27.

All the pupils will equal $\frac{5}{3}$ of 27 = 45 Ans.

DECIMAL FRACTIONS

.76 of the population equals 4066. Therefore .01 of the population equals $4066 \div 76$ (or $\frac{1}{76}$ of 4066).

The total population will equal 100 times ($4066 \div 76$) or 5350 Ans.

A shorter explanation is: .76 of the population equals 4066; therefore the population equals $\frac{100}{76}$ of 4066, or $4066 \div .76 = 5350$ Ans.

ORAL EXERCISES

62. 1. 4 is what part of 6? Of 8? Of 12?
(Give the answers in .00's.)

2. 7 is what part of 8? Of 10? Of 20?

3. 5 is what part of 15? Of 25? Of 35?
Of 45?

4. 6 is what part of 30? Of 36? Of 48?
Of 60?

5. 8 is what part of 12? Of 20? Of 32?
Of 100?

6. $\frac{1}{4}$ is what part of $\frac{1}{2}$? Of $\frac{3}{4}$? Of $\frac{7}{8}$? Of $1\frac{1}{4}$?

WRITTEN PROBLEMS

(Give answers in .00's.)

63. 1. A man spent \$24 out of an allowance of \$420. What part did he spend? What part did he have left?

2. There are 756 pupils present in a school of 800. What part of the school is absent?

3. A farmer having 720 sheep, sold 600 of them. What part did he sell? What part of the flock did he keep?

4. If he had sold 40 sheep more than he did, what part of the flock would have been left?

5. 4550 grains is what part of an avoirdupois pound?

ORAL EXERCISES

(Give answers in .00's.)

64. 1. 4 in. = what part of a foot?
2. 8 hr. = what part of a day?
3. 24 in. = what part of a yard?
4. 6 in. = what part of 2 ft.?
5. 12 oz. = what part of a pound (avoir.)?
6. 75 lb. = what part of a hundredweight?
7. 1500 lb. = what part of a ton?
8. 15 min. = what part of an hour?
9. 40 min. = what part of 2 hr.?
10. 24 qt. = what part of a bushel?

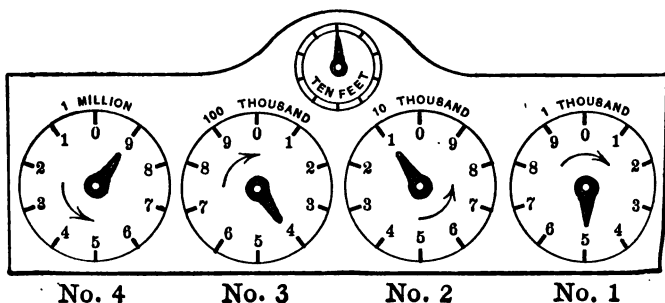
ORAL PROBLEMS

65. 1. Mary is 16 yr. old. John's age is .80 of Mary's age. How old is John?
2. A farm of 60 acres contains .15 of woodland. How many acres of woodland?
3. 14 pupils are absent; this is .05 of the number on register. How many pupils are on register?
4. My coat cost \$18, which is $.12\frac{1}{2}$ more than the cost of my dress. Find the cost of the dress.
5. A 50-cent knife was sold for 55 ct. How many .00's were gained?

METERS

66. A meter is an instrument used for measuring and recording the quantity, force, and pressure of a fluid passing through it. The meters in general use are gas meters, water meters, and electric light meters.

The dials of most meters are read in the decimal scale. The indicator on an ordinary gas meter looks like the following diagram.



Dial No. 1 in a complete revolution records 1000 cu. ft. of gas. Each division on that dial represents 100 cu. ft. of gas used. The hand points to 5. This indicates that 500 cu. ft. of gas have been used since the last thousand was recorded.

Whenever the hand of dial No. 1 makes a complete revolution, the hand of dial No. 2 moves from one division to the next. The hand of dial No. 1 now represents 500 cu. ft. When the hand of dial No. 1 completes its revolution, the hand of dial No. 2 will reach division 2 on its own dial.

In the same way, an entire revolution on dial No. 2 will cause the hand of dial No. 3 to move through one division.

Ten complete revolutions of the hand of any dial will produce one complete revolution of the hand of the dial next on the left.

EXERCISES

1. Read the record on dial No. 1.
2. On dial No. 2.
3. On dial No. 3.
4. On dial No. 4.
5. Read the entire number of cubic feet of gas used, as recorded on the face of the meter.
6. Where would the hands be to represent the use of 629,200 cu. ft.?
7. On December first, a gas meter indicated 169,300 cu. ft.; on January first the meter recorded 176,200 cu. ft. What was the amount of the bill at 80 ct. per M.?

Process

Present state of meter 176,200

Previous state of meter 169,300

Amount used 6900 cu. ft.

6900 cu. ft. = 6.900 M (or thousands)

.80
\$ 5.52 amount of bill.

8. Make out gas bills for the following:

Present state of meter, 268,400; previous state of meter, 236,900. Rate, $87\frac{1}{2}$ ct. per M.

Previous state of meter, 187,400; present state of meter, 201,000. Rate, \$1 per thousand.

BILLS AND STATEMENTS

67. The following shows the general form of bill:

ALBANY, N.Y., Jan. 9, 1911.

MR. DONALD STUART,
72 State St.

Bought of JOHN G. MYERS CO.
North Pearl Street

TERMS: Cash

Telephone, Main 400

1910							
Dec.	11	6 yd. Ribbon	.43	2	58		
"	16	1 doz Napkins		3	50		
"	22	6 Handkerchiefs	.35	2	10	7	18

Received payment, Jan. 11, 1911

JOHN G. MYERS Co.

per TOWNSEND

A bill should show:

1. The place and date of sale.
2. The name and address of the buyer, or debtor.
3. The name and address of the seller, or creditor.
4. The quantity, name, and price of each item.
5. The total amount or footing.

The creditor sends the bill to the debtor. When the debtor pays the bill, he usually gets from the creditor a receipt or acknowledgment that the bill has been paid. In most cases acknowledgment is made on the bill itself by the creditor or his representative, who signs his name after the words "Received Payment" or "Paid." If a representative of the creditor receives the money, he usually signs his employer's name and then his own name preceded by the word "per" or "by."

The following is a model form of statement.

CHICAGO, ILL., Mar. 28, 1911.

MR. ALFRED R. REEVES

Dayton, Ohio

In account with F. R. MANSFIELD

1911		Dr.				
Feb.	1	To acct. rendered	568	40		
Feb.	8	" Mdse	605	80		
"	15	" "	1850		3024	20
		Cr.				
Feb.	9	By cash	550			
"	29	" "	600			
Mar.	1	" "	1000		2150	
		Balance			874	20

The above is Mr. Mansfield's statement to Mr. Reeves. It shows that Mr. Reeves is Mr. Mansfield's debtor for the various amounts represented

by his purchases. These amounts are placed in the debit account, marked Dr.

The statement also shows that Mr. Reeves is Mr. Mansfield's creditor for the various amounts represented by his payments. These amounts are placed in the credit account, marked Cr.

FORM OF CHECK

<i>No. 723</i>	<i>Albany, N.Y.,---Jan 11,---1911.</i>
Commercial Bank of Albany STATE STREET	
<i>Pay to the order of-----John G. Myers Co.-----</i>	
<i>Eight $\frac{18}{100}$-----Dollars,</i>	
<i>\$ 8 $\frac{18}{100}$</i>	<i>-----Donald Stuart.-----</i>

Reverse Side of Check

I. INDORSEMENT FOR DEPOSIT IN A BANK

<i>For deposit</i> <i>John G. Myers Co</i>	
---	--

Reverse Side of Check

II. INDORSEMENT FOR TRANSFER FROM ONE OWNER TO ANOTHER

<i>Pay to the order of</i> <i>Gatriek Campbell.</i> <i>John E. Meyers Co.</i>

EXERCISES

Make out, foot, and receipt the following bills. Supply necessary dates, names, and addresses.

1. G. C. Hunt bought of Stern Bros.: 1 pr. shoes at \$5.50; 6 pr. stockings @ 3 pr. for a dollar; 1 doz. towels for \$2.75.

2. The Baker & Taylor Co. sold to you: 1 copy Lady of the Lake, \$.75; 3 copies Shakespeare @ \$2.25; 1 set Dickens, \$27.

3. Turkey $11\frac{1}{2}$ lb. @ 24 ct.; Chickens $8\frac{3}{4}$ lb. @ 18 ct.; $9\frac{1}{2}$ lb. Roast Beef @ 22 ct.

4. A plumber repairs a leak in a water pipe in Mrs. Brown's house. He charges for $3\frac{1}{2}$ days' work for a man and his helper; and \$1.75 for materials used.

DENOMINATE NUMBERS

68. Numbers like 8 marbles and 15 horses are **concrete numbers**. Numbers like 8 and 15 are **abstract numbers**. Numbers like 10 bushels, \$8, or 15 inches, in which the unit of measurement has been established by law or by general usage, are **denominate numbers**.

A number consisting of two or more units of the same kind is a **compound denominate number**, as 1 yd. 3 in.; 7 lb. 4 oz.; 6 da. 2 hr. 40 min.

Denominate numbers are divided into tables of weights, measures, and values.

Tables of Weights, Measures, and Values

NOTE. All the tables of weights, measures, and values (except those of the Metric System) are here given for reference. The most important tables are marked *. All or nearly all the steps of tables marked * should be memorized.

Some of the denominations are no longer in general use. The examples selected for application are chosen from the tables and terms now in frequent use.

It is suggested that teachers reduce the work in reduction, addition, subtraction, multiplication, and division of denominate numbers to a minimum. Long examples and large numbers should be avoided. The methods of solution should conform to business usage.

69. Tables of Weights

Avoirdupois Weight *

7000 (Troy) grains (gr.)	= 1 pound (lb.)
16 ounces (oz.)	= 1 pound (lb.)
100 pounds	= 1 hundredweight (cwt.)
2000 pounds	= 1 ton (T.)
2240 pounds	= 1 gross or long ton (L.T.)

An ounce avoirdupois weighs $437\frac{1}{2}$ Troy grains

Troy Weight

24 grains	= 1 pennyweight (dwt.)
20 pennyweights	= 1 ounce (oz.)
12 ounces	= 1 pound (lb.)
5760 grains	= 1 pound

An ounce Troy weighs 480 Troy grains

Apothecaries' Weight

20 grains (gr.)	= 1 scruple (sc. or \mathfrak{S})
3 scruples	= 1 dram (dr. or \mathfrak{D})
8 drams	= 1 ounce (oz. or \mathfrak{Z})
12 ounces	= 1 pound (lb.)

Miscellaneous Weights

60 pounds (lb.)	= 1 bushel of potatoes
32 pounds	= 1 bushel of oats
60 pounds	= 1 bushel of wheat
56 pounds	= 1 bushel of corn (shelled)
56 pounds	= 1 bushel of rye
48 pounds	= 1 bushel of barley
80 pounds	= 1 bushel of coal
200 pounds	= 1 barrel of pork
196 pounds	= 1 barrel of flour
56 pounds	= 1 firkin of butter
62½ pounds	= 1 cubic foot of water

70. Tables of Measures**Areas or Surface Measures**

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
30¼ square yards	= 1 square rod (sq. rd.)
160 square rods	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)
	or section
36 square miles	= 1 township

Length or Linear Measure *

12 inches (in.) = 1 foot (ft.)

3 feet

or

36 inches

= 1 yard (yd.)

5½ yards

or

16½ feet

= 1 rod (rd.)†

320 rods

or

1760 yards

= 1 mile

or

5280 feet

39.37 inches = 1 meter (m.)

.6214 mi. or $\frac{5}{8}$ mile = 1 kilometer (km.)**Table of Volume or Contents**

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)

27 cubic feet = 1 cubic yard (cu. yd.)

128 cubic feet = 1 cord ‡

† The rod is sometimes called a perch or a pole.

‡ A cord of wood is a pile of wood 4 ft. wide, 4 ft. high, and 8 ft. long. $4' \times 4' \times 8' = 128$ cu. ft. A cord foot is a straight pile of wood $4' \times 4' \times 1'$.

Surveyors' Linear Measure

7.92 inches = 1 link (li.)

66 feet or 100 links = 1 chain (ch.)

80 chains = 1 mile (mi.)

Surveyors' Surface Measure

625 square links (sq. li.) = 1 square rod

16 square rods = 1 square chain

10 square chains = 1 acre (A.)

Apothecaries' Liquid Measure

60 drops = 1 fluid dram (f. 3)

8 fluid drams = 1 fluid ounce (f. 3)

16 fluid ounces = 1 pint (O.)

8 pints = 1 gallon (Cong.)

Miscellaneous Measures of Length or Surface

4 inches = 1 hand (used in measuring
the height of horses)

6 feet = 1 fathom (used in measuring
the depth of water)

$1\frac{1}{8}$ miles = 1 knot or nautical mile

100 square feet = a square (of roof, pavement)

Measure of Time *

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
7 days	= 1 week (wk.)
365 days	= 1 year (yr.)
366 days	= 1 leap year †

Days by Months

January	} 31 days
March	
May	
July	
August	
October	
December	} 30 days
April	
June	
September	
November	
February	{ 28 days in a common year 29 days in a leap year

† According to the ordinary calendar, every year whose date number is divisible by 4 is a leap year, unless the date number ends in two ciphers; in this case, only years whose date number is divisible by 400 are leap years; 1900 was not a leap year; 2000 will be a leap year.

Measure of Angles

60 seconds (") = 1 minute (')

60 minutes = 1 degree (°)

90 degrees = 1 right angle (rt. ∠)
or 1 quadrant

360 angle degrees = 4 right angles

360 arc degrees = 1 circumference

Miscellaneous Tables

12 things = 1 dozen (doz.)

12 dozen or
144 things } = 1 gross (gro.)

12 gross or
1728 things } = 1 great gross (gt. gro.)

20 things = 1 score (sc.)

24 sheets = 1 quire (qr.)

20 quires or
480 sheets } = 1 ream (rm.)

Liquid Measure *

4 gills (gi.) = 1 pint (pt.)

2 pints = 1 quart (qt.)

4 quarts = 1 gallon (gal.)

A liquid gallon contains 231 cu. in.

Dry Measure *

2 pints (pt.)	= 1 quart (qt.)
8 quarts	= 1 peck (pk.)
4 pecks or 32 quarts }	= 1 bushel (bu.)

The standard or Winchester bushel is a cylinder $18\frac{1}{2}$ in. in diameter and 8 in. deep. It contains about 2,150.42 cu. in. or about $\frac{5}{4}$ cu. ft.

71. Tables of Values**U. S. Money ***

10 mills (m.)	= 1 cent (ct.)
10 cents	= 1 dime (d.)
10 dimes or 100 cents }	= 1 dollar (\$)
10 dollars	= 1 eagle
20 dollars	= 1 double eagle

English Money

4 farthings (far.)	= 1 penny (d.)
12 pence	= 1 shilling (s.)
20 shillings	= 1 pound (£)
21 shillings	= 1 guinea (G.)

Table of Money Equivalents

COUNTRY	STANDARD	APPROXIMATE VALUE	EXACT VALUE
United States and Canada	} Dollar	\$1.00	\$1.00
Great Britain		Pound (sterling) 5.00	4.8665
Germany	Mark	.25 (4 to the dollar)	.238
France	Franc	.20 (5 to the dollar)	.193
Russia	Ruble	.75 (4 to 3 dollars)	.772
Austria	Crown	.20 (5 to the dollar)	.203
Italy	Lira	.20 (5 to the dollar)	.193

All these countries except Great Britain use a decimal currency.

FORMS OF REDUCTION OF DENOMINATE NUMBERS

72. 1. Reduction to a lower denomination. (Reduction Descending.)

(a) Change 6 gal. to quarts.

Process

4 qt. in 1 gal.
6
24 qt. in 6 gal.

Explanation. There are 4 qt. in 1 gal.
In 6 gal. there must be 6×4 qt., or
24 qt.

(b) Change 6 gal. 3 qt. to quarts.

Process

4 qt. in 1 gal.
6
24 qt. in 6 gal.
3 qt.
27 qt. in 6 gal. 3 qt.

Explanation. There are 4 qt.
in 1 gal. In 6 gal. there are
24 qt. 24 qt. plus 3 qt. = 27 qt.

(c) Change $\frac{4}{5}$ of a day to lower denominations.

$\frac{4}{5}$ of 24 hr. = $\frac{96}{5}$ = $19\frac{1}{5}$ hr.

$\frac{1}{5}$ of $\frac{12}{60}$ min. = 12 min.

19 hr. 12 min. *Ans.*

(d) Change .175 of a year to months and days.

12 mo. 30 da.

.175 .1
2.100 mo. 3 da. 2 mo. 3 da. *Ans.*

2. Reduction to a higher denomination. (Reduction Ascending.)

(a) Change 74 in. to feet.

<p>Process</p> $12 \overline{) 74} \text{ in.}$ $6 \text{ ft. } 2 \text{ in.}$	<p>Explanation. 1 in. = $\frac{1}{12}$ ft. Therefore, the number of feet = $\frac{1}{12}$ of the number of inches.</p>
---	---

(b) Change 17,640 ft. to miles.

<p>Process</p> $5280 \overline{) 17640}$ $\underline{1584}$ $\frac{180}{528} = \frac{15}{44}$	<p>Explanation. 1 ft. = $\frac{1}{5280}$ mi. Therefore, the number of miles = $\frac{1}{5280}$ of 17,640 = $3\frac{1}{4}$ mi. <i>Ans.</i></p>
--	---

(c) Change £ 16 7s. 6d. to £ and a decimal of a pound.

<p>Process.</p> $12 \overline{) d. 6.0}$ $20 \overline{) s. .7 \times 50}$ $\text{£ } 16.375.$

(d) Change 5 T. 12 cwt. 50 lb. to tons and a common fraction of a ton.

<p>Process</p> $\frac{12\frac{1}{2}}{20} = \frac{5}{8}$	<p>Explanation</p> <p>50 lb. = $\frac{1}{2}$ cwt. 12 cwt. 50 lb. = $12\frac{1}{2}$ cwt. $12\frac{1}{2}$ cwt. = what part of 20 cwt. (or 1 T.)? $12\frac{1}{2}$ cwt. = $\frac{12\frac{1}{2}}{20}$ of a ton = $5\frac{5}{8}$ T. <i>Ans.</i></p>
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EXERCISES ON IMPORTANT TABLES ONLY

73.

Avoirdupois Weight

ORAL EXERCISES

1. How many pounds in half a ton?
2. 1500 lb. is what part of a ton?
3. How many tons in 7000 lb.?
4. $.37\frac{1}{2}$ of a ton equals how many pounds?
5. Change 1250 lb. to a fraction of a ton.

WRITTEN EXERCISES

1. $17\frac{3}{8}$ tons equals how many pounds?
2. Change 19,740 lb. to tons.
3. How many pounds in $20\frac{1}{4}$ long tons?
4. 1350 lb. = how many .00's of a ton?
5. What is the difference in pounds between $8\frac{1}{2}$ long tons and $8\frac{1}{2}$ ordinary tons?

ORAL PROBLEMS

1. How many 40-lb. bags of coal can be obtained from half a ton of coal?
2. A wheelbarrow holds 200 lb. of gravel. How many times will the barrow be filled in carting 2 tons of gravel?

3. A table weighs 40 lb. The weight of 25 such tables is what part of the weight of a ton?

4. At $\frac{1}{2}$ ct. a pound, what is the cost of $\frac{1}{4}$ of a ton of ice?

5. How many 25-lb. bags of flour can be filled from a carload weighing 2 tons?

WRITTEN PROBLEMS

1. Coal can be bought at \$6.50 a ton, or in 80-lb. bags at 40 ct. a bag. How much does a man lose if he buys 150 bags during the year?

2. A dealer pays \$2.75 a ton (long) for ice. He sells the ice at 35 ct. a hundred pounds. What does he gain on the sale of 18 tons of ice? (Allow a loss of 240 lb. to the ton for melting, etc.)

3. A man bought 3 tons of iron scraps. He sold 1250 lb. How many .00's of his stock did he sell?

4. A 15-ton shipment of canned goods was packed in cases each containing 2 doz. 1-lb. cans. How many cans were used? How many cases?

5. Freight costs 35 ct. a hundred pounds. What is the cost of a shipment of $18\frac{3}{4}$ tons?

74.

Long Measure

ORAL EXERCISES

1. Change 45 in. to yards.

2. How many feet in half a mile?

3. 180 in. = how many yards?

4. How many feet in 64 in.?
5. 27 in. = what part of a yard?

WRITTEN EXERCISES

1. How many feet in $10\frac{2}{3}$ mi.?
2. Change 231 in. to yards.
3. How many yards in $\frac{7}{8}$ mi.?
4. 880 ft. = how many .00's of a mile?
5. $.17\frac{1}{2}$ of a yard = how many inches?

ORAL PROBLEMS

1. How many yards of ribbon are needed to bind a sofa pillow 20 in. square, allowing 4 in. for turning corners?
2. How many tape loops each containing 6 in. can be cut from a roll of tape containing $3\frac{1}{2}$ yd.?
3. How many rolls of wire 60 ft. in length will be required for a single telephone wire a mile long?
4. At 30 ct. a yard, what is the cost of 24 in. of ribbon?
5. A race course is $\frac{1}{8}$ of a mile long. After a boy has run 200 yd., how much farther must he run to finish the course?

WRITTEN PROBLEMS

1. How many yards of ribbon are needed to make 15 loops, each containing 8 in.?

2. An aëroplane is $1\frac{3}{4}$ miles above the surface of the earth. How many feet must it still rise to reach an altitude of 10,000 ft.?

3. A wheel has a circumference of 120 in. How many times will it revolve in covering 225 yd.?

4. A road is $7\frac{1}{2}$ miles long. The land on one side of the road is divided into plots, each plot being 75 ft. long. How many plots are there?

5. How many yards of fence are needed to inclose a square garden, $62\frac{1}{2}$ ft. long?

75. Time Measure

ORAL EXERCISES

1. 15 min. is what part of 2 hr.?
2. 7 mo. 15 da. is how much less than 1 yr.?
3. How many hours in $5\frac{1}{2}$ da.?
4. 8 mo. = how many .00's of a yard?
5. How many minutes in 330 sec.?

WRITTEN EXERCISES

1. How many days between Jan. 4 and June 2?
2. Add 90 da. to March 22. What is the date?
3. What is the difference in years and months between October, 1876, and May, 1903?
4. How many months in $16\frac{2}{3}$ yr.?
5. 45 min. = what part of 3 hr.?

ORAL PROBLEMS

1. A clock strikes every 15 min. How many times will it strike beginning at 8.30 A.M. and ending at noon?

2. A boy saves 3 ct. every day. How much will he save during the month of March?

3. How many days during June, July, and August?

4. A boy earns \$4.50 per week. What will he lose by being absent for a day?

5. A boy requires 3 hr. in which to complete his work. What part of it can be completed in 20 min.?

WRITTEN PROBLEMS

1. A newsboy makes a profit of 40 ct. a day. What was his profit during January, February, and March, 1911?

2. How many doses of medicine will a patient receive, who takes his medicine every 40 min. beginning at 9 A.M. and ending at 9 P.M.?

3. John was born on Oct. 12, 1898. How old is he to-day?

4. How old was he on Feb. 7, 1909?

5. A man borrowed a sum of money on Oct. 17, 1910. He kept it 95 da. On what date did he return the money?

76.

Liquid Measure

ORAL EXERCISES

1. How many quarts in 72 gills?
2. $2\frac{1}{2}$ qt. equals what part of a gallon?
3. Change 17 pt. to gills.
4. How many pints in $.62\frac{1}{2}$ of a gallon?
5. 10 gal. equals how many pints?

WRITTEN EXERCISES

1. How many pints in 7 gal. 3 qt.?
2. 176 pt. equals how many gallons?
3. 3 gi. equals how many .00's of 3 gal.?
4. $4\frac{1}{2}$ gal. equals how many times $4\frac{1}{2}$ pt.?
5. 2 qt. 1 pt. equals how much less than 1 gal.?

ORAL PROBLEMS

1. How many quart bottles can be filled from $6\frac{1}{2}$ gal. of oil?
2. At 7 ct. a $\frac{1}{2}$ pint, what is the cost of 2 qt. of cream?
3. A furniture polisher uses 1 pt. of polish every day. How many gallons of polish will he use in 28 days?
4. A cup of cider contains half a pint. How many cups of cider can be drawn from a 10-gal. keg of cider?
5. How much must be paid for 4 gal. of ice cream at 30 ct. a quart?

WRITTEN PROBLEMS

1. How many bottles, each containing $1\frac{1}{2}$ pt. of oil, can be filled from $37\frac{1}{2}$ gal.?
2. Ice cream costs a dealer \$1 a gallon. He sells it in portions, eight to the quart. He charges 5 ct. a portion. What is his gain on $6\frac{1}{2}$ gal.?
3. Cream costs 14 ct. a half pint. What is the bill for $\frac{3}{4}$ of a gallon of cream?
4. From a can containing 15 gal. of milk, a farmer sells 3 qt., 5 qt., $7\frac{1}{2}$ qt., and 10 qt. How much milk is not sold?
5. A family uses $3\frac{1}{2}$ qt. of milk daily. At that rate, how many gallons of milk will it use during the months of March, April, May, June, and July?

77.

Dry Measure

ORAL EXERCISES

1. 12 qt. = what part of a bushel?
2. How many pints in 1 pk.?
3. 3 pk. = how many .00's of a bushel?
4. Change 58 qt. to pecks?
5. From 1 bushel take 1 pk. 2 qt.

WRITTEN EXERCISES

1. How many pints in $2\frac{1}{2}$ qt.?
2. $2\frac{3}{4}$ bu. = how many quarts?
3. 2 qt. 1 pt. = what part of a peck?

4. How many bushels in 149 pk.?
5. Change $30\frac{1}{2}$ bu. to quarts.

ORAL PROBLEMS

1. A farmer uses 12 qt. of oats a day. How many bushels, at that rate, will he use in 8 days?
2. A farmer uses 12 qt. of oats a day. At that rate, how long will 10 bu. of oats last?
3. Apples cost \$2 per bushel. They were sold at 10 ct. a quart. What was the gain per bushel?
4. If 2 qt. of apples yield 1 cup of apple juice, how many cups of juice will 1 bushel of apples yield?
5. What will 2 pk. of carrots cost at 10 ct. a quart?

WRITTEN PROBLEMS

1. A bushel contains 2150.42 cu. in. How many cubic inches in 1 peck?
2. A boy earns 2 ct. a pint for gathering chestnuts. What does he earn for gathering $10\frac{7}{8}$ bu.?
3. I can buy potatoes for 70 ct. a bushel, or else at the rate of 2 qt. for 7 ct. I prefer to buy at the latter price. What is my loss at the end of 12 weeks if I buy 6 bu. of potatoes each week?
4. A farmer puts $72\frac{3}{4}$ bu. of pears into quart boxes. He packs them in crates, each crate containing 2 doz. boxes. How many crates are needed?

5. Apples cost \$1.28 per bushel. For what must I sell them per quart to double my money?

ORAL EXERCISE I

78. Change at sight to the next lower denomination:

- | | | |
|------------------|------------------------------|-------------------------|
| 1. 7 ft. | 7. \$18 | 13. $6\frac{1}{4}$ cwt. |
| 2. 9 T. | 8. £ 5 | 14. $5\frac{1}{4}$ pk. |
| 3. 10lb.(avoir.) | 9. 2 gro. | 15. 15 min. |
| 4. 6 hr. | 10. 7 doz. | 16. $3\frac{1}{2}$ da. |
| 5. 5 gal. | 11. $6\frac{2}{3}$ yd. | 17. 5 leap yr. |
| 6. 7 bu. | 12. $5\frac{1}{2}$ lb.(Troy) | 18. 6s. |

ORAL EXERCISE II

Change at sight to the next higher denomination:

- | | |
|---------------------|-----------------|
| 1. 36 in. | 9. 240 hr. |
| 2. 40 pk. | 10. 640 rd. |
| 3. 16 pt. | 11. 270 cu. ft. |
| 4. 200 lb. (avoir.) | 12. 210 da. |
| 5. 300 min. | 13. 3,000 sec. |
| 6. 48 qt. (liquid) | 14. 36 sq. ft. |
| 7. 60 doz. | 15. 200 dwt. |
| 8. 100s. | 16. 75 ft. |

ORAL EXERCISE III

Change at sight to the lowest denomination named:

- | | |
|----------------------|---------------------------|
| 1. 2 ft. 5 in. | 9. £3 7s. |
| 2. 5 yd. 11 in. | 10. 9s. 6d. |
| 3. 3 bu. 1 pk. | 11. \$1 1 d. 1 ct. |
| 4. 10 bu. 3 qt. | 12. 5 lb. 7 oz. (Troy) |
| 5. 5 gal. 2 qt. | 13. 100 sq. yd. 4 sq. ft. |
| 6. 5 hr. 14 min. | 14. 20 yd. 3 ft. |
| 7. 6 min. 25 sec. | 15. 1 yr. 8 mo. 5 da. |
| 8. 1 yd. 1 ft. 1 in. | 16. 7 bu. 5 qt. |

ORAL EXERCISE IV

Change at sight to the denomination required:

1. 75 in. to yards.
2. 42 oz. to pounds (apoth.).
3. 50 sheets to quires.
4. 300 things to gross.
5. 75d. to shillings.
6. 63 qt. to pecks.
7. 54 da. to weeks.
8. 97 da. to months.
9. 10,560 ft. to miles.
10. 215 pt. to gallons.
11. 75 hr. to days.
12. 30 oz. (Troy) to pounds.

WRITTEN EXERCISE I

79. 1. Change 3 cu. ft. to cubic inches.
 2. Change $7\frac{1}{2}$ mi. to feet.
 3. Change $12\frac{1}{5}$ T. to pounds.
 4. Change £10 8s. to shillings.
 5. How many pencils in 3 gro. 7 doz.?
 6. How many quarts in 5 bu. 3 pk.?
 7. How many pints in 8 gal. 1 qt.?
 8. How many pounds in 32.7 bu. of potatoes?
 9. How many pounds in 17 firkins of butter?
 10. How many days in $17\frac{2}{3}$ months?

WRITTEN EXERCISE II

Change to required denominations:

1. 235 in. to yards, feet, and inches.
2. 562 min. to hours and minutes.
3. 725 sheets to reams, quires, and sheets.
4. 12,806 lb. to tons and pounds.
5. 1376 pints to bushels, pecks, and quarts.
6. 7 mo. 15 da. to decimal of a year.
7. 220 ft. to a fraction of a mile.
8. $\frac{1}{8}$ mi. to feet.
9. $\frac{3}{5}$ T. to pounds.
10. .625 A. to square rods.
11. 1.7 lb. (Troy) to grains.
12. 7 yr. 5 mo. to days.

13. 3 qt. to a fraction of 3 bu.

14. 15s. to .00's of £1.

ORAL PROBLEMS

80. 1. At 5 ct. each, find the cost of 5 doz. oranges.

2. How many $\frac{1}{2}$ -pt. bottles are required to hold 1 gal. of oil?

3. A boy runs a city block in 2 min. How many blocks can he run in $\frac{1}{2}$ hr.?

4. Paper costs 35 ct. a quire, but 2 ct. a sheet. How much is saved by buying 2 quires?

5. How many 8-in. loops can be made out of 2 yd. of ribbon?

6. A vender buys peanuts at \$5 per bushel and sells them at 10 ct. per pint. How much does he gain on 2 bu.?

7. Candles cost 95 ct. per gross. They retail at the rate of 3 for 5 ct. Find the gain on each gross.

8. Potatoes cost 85 ct. a bushel. They were sold at 3 ct. a lb. Find the gain or loss on 10 bu.

9. A man sold $22\frac{1}{2}$ lb. out of a firkin of butter. How many pounds remained in the tub?

10. A child must take his medicine every 15 min. How many doses of medicine will he take from noon until half past four?

WRITTEN PROBLEMS

1. How many rings weighing $3\frac{1}{2}$ dwt. each can be made out of $7\frac{1}{2}$ oz. of gold?
2. At 40 ct. per hundredweight, find the cost of 725 lb. of ice.
3. A drug clerk puts 4 gr. of quinine into each capsule. How many capsules can be filled from .25 of an oz. of the drug?
4. At 75 ct. per running foot, what must be paid for putting a fence entirely around a square field measuring 25 yd. to the side?
5. Which is heavier, and how much, a pound of butter or a pound of gold? Give the answer in grains.
6. Find the cost of 12,450 shingles at \$3.75 per M. (M stands for 1000.)
7. Cost of 8260 boards at \$24 per M?
8. Cost of 22 A. at \$150 per acre?
9. Find the value of $\frac{1}{2}$ doz. silver spoons, each weighing 3 oz., silver being worth \$1.12 $\frac{1}{2}$ per ounce.
10. A pile of wood 30 ft. long 4 ft. wide contains 6 cords. How high is the pile? (See p. 95.)

ADDITION OF DENOMINATE NUMBERS

Addition of Concrete Units of Same Name

81. 1. Add 7 mo., 3 mo., 9 mo., 8 mo., 6 mo.

Process

mo.

7**3****9****8****6****33** mo.

Changing 33 mo. to years and months, the answer is 2 yr. and 9 mo.

Addition of Compound Numbers**2. Add:**

£	s.	d.
2	8	4

1	9	7
---	---	---

3	—	5
---	---	---

6	16	2
---	----	---

13	14	6
----	----	---

18d.=1s. and 6d., 34s.=£ 1 and 1s.

Ans. £ 13 14s. 6d.**Add:****3.**

bu.	qt.
7	5

9	14
---	----

8	20
---	----

4.

ft.	in.
7	4

8	3
---	---

9	11
---	----

5.

ml.	ft.
7	2,900

6	3,728
---	-------

5	4,169
---	-------

6.

hr	min.	sec.
7	24	36

8	16	42
---	----	----

10	38	50
----	----	----

9	48	52
---	----	----

7.

£	s.	d.
1	8	7

3	13	6
---	----	---

7	18	9
---	----	---

4	16	11
---	----	----

8.

lb.	oz.	dwt.
3	5	11

8	9	15
---	---	----

6	8	8
---	---	---

4	3	16
---	---	----

SUBTRACTION OF DENOMINATE NUMBERS

Process		Explanation.
	gal. qt.	3 qt. cannot be taken from 1 qt. Take
82. 1. From	9 1	1 gal. (leaving 8 gal.) and
take	2 3	change the 1 gal. to 4 qt. 3 qt.
	6 2	from 4 qt. leaves 1 qt. 1 qt.
		+ the 1 qt. in the minuend, is
		2 qt. 2 gal. from 8 gal. = 6 gal.

2. Find the difference in time between March 24, 1876, and Dec. 11, 1902.

Process			
yr.	mo.	da.	
	11		
1902	<u>12</u>	(30)11	March is the 3d mo.
1876	3	24	December is the 12th mo.
26	8	17	26 yr. 8 mo. 17 da. <i>Ans.</i>

3. Find the number of days from Aug. 24 to Nov. 12.

Process		Explanation.
Aug. 7		August has 31 da.
Sept. 30		From the 24th to the end of the month there are 7 da. September, full month,
Oct. 31		30 da. October, full month, 31 da.
Nov. 12		November, 12 da. Total, 80 da.
	<u>80</u> da.	

4. From 3 yr. take 1 yr. 8 mo. 16 da.

5. From 1 T. take 17 cwt. 75 lb.

6. Find the difference in time between July 20, 1903, and Aug. 17, 1910.

7. If you cut a piece of ribbon 5 ft. 7 in. long from a roll of ribbon containing 10 yd., how much ribbon is left?

8. Washington was born on Feb. 22, 1732; he died on Dec. 13, 1799. Find his exact age at the time of his death. How long is it since he was born?

EXERCISES

Find the difference between the following dates:

1. Jan. 5, 1862, and to-day.
2. July 4, 1776, and to-day.
3. Oct. 12, 1492, and July 4, 1776.
4. April 30, 1789, and March 4, 1861.
5. Dec. 21, 1620, and March 4, 1789.

MULTIPLICATION OF DENOMINATE NUMBERS

Multiplication of One Denomination Only

83. 1. Multiply 12 qt. (dry measure) by 7.

Process

$$\begin{array}{r} 12 \text{ qt.} \\ 7 \\ \hline 32 \overline{)84} 2 \\ 64 \\ \hline 20 \\ 32 \overline{)20} = \frac{5}{8} \end{array}$$

32 qt. = 1 bu.

2 bu. 20 qt.
or
2 $\frac{5}{8}$ bu

Explanation. $7 \times 12 \text{ qt.} = 84 \text{ qt.}$ There are 32 qt. in 1 bu.; therefore, the number of bushels equals $\frac{1}{32}$ of 84, or $84 \div 32 = 2\frac{5}{8}$ bu., or 2 bu. 20 qt. *Ans.*

Multiplication of a Compound Number

2. Multiply 11s. 4d. by 25.

Process

s	d.	
11	4	
	25	
£	s.	d.
14	3	4

Explanation. $25 \times 4d. = 100d. = 8s. 4d.$
 $25 \times 11s. = 275s. \quad 275s. + 8s. = 283s. =$
 $£14 \ 3s.$
 $£14 \ 3s. \ 4d. \quad Ans.$

PROBLEMS

3. How much tape is needed for binding 2 doz. curtains, if each one takes 7 yd. 19 in. of tape?

4. A wheel is 8 ft. 4 in. in circumference. How far will it go in 17 revolutions?

5. English gloves are worth £3 9s. 6d. per doz. Find the cost of 6 doz.

6. One panel takes 1 ft. 4 in. of board. How much wood is needed to make 20 panels?

7. One gold ring weighs 3 dwt. 18 gr. Find the weight of 1 doz. rings of the same style and size.

8. Allowing 3 hr. 40 min. to make an apron, how long will it take to make 20 aprons? Change the answer to days, allowing 9 working hours to a day.

DIVISION OF DENOMINATE NUMBERS

84. 1. Divide 14 pk. 6 qt. by 4.

Process			Explanation.
pk.	qt.	pt.	$\frac{1}{4}$ of 14 pk. = 3 pk.
4)14	16+6	4+0	+ 2 pk. remainder. Change the
3+2	5+2	1	remainder to the next denomina-
			tion. 2 pk. = 16 qt. 16 qt. + the
			6 qt. in the dividend = 22 qt. $\frac{1}{4}$ of

22 qt. = 5 qt. + 2 qt. remainder. Change the remainder to the next denomination. 2 qt. = 4 pt. $\frac{1}{4}$ of 4 pt. = 1 pt. 3 pk. 5 qt. 1 pt. *Ans.*

2. Divide 7 yd. 2 ft. 8 in. by 4.

3. Divide £4 6s. 6d. by 3.

4. If 6 doz. lace collars cost £8 12s. 4d., what is the cost of 1 doz.?

5. How long is each bow of ribbon if I make 4 bows out of 7 yd. 16 in. of ribbon?

6. A wheel covered 75 yd. 2 ft. in 20 revolutions. What was the length of the circumference of the wheel?

MEASUREMENT OF SURFACES

AREAS

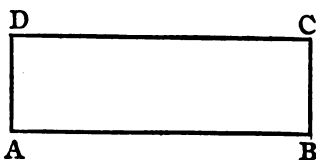
85. Surfaces are measured by Surface Measure. Land is more frequently measured by Surveyors' Surface Measure. (See pages 94 and 96.)

Rectangles

A **rectangle** is a plane figure bounded by four straight lines and having four right angles. The figure *ABCD* is a rectangle.

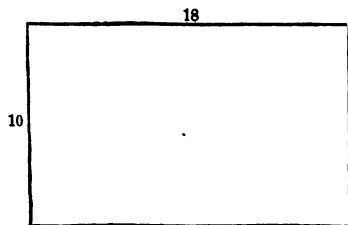
AB (or *DC*) is called the **base** of the rectangle.

AD (or *BC*) is called the **height** or **altitude** of the rectangle.



What is the area of a rectangle 18 yd. long and 10 yd. wide?

$$10 \times 18 \text{ sq. yd.} = 180 \text{ sq. yd.}$$

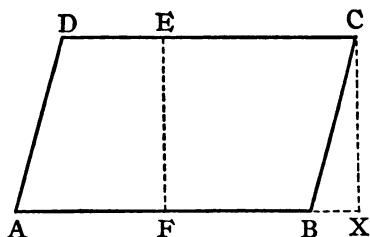


Explanation. Divide the width into 10 equal parts; each strip, or part, will be 1 yd. wide. What is the area of a strip 18 yd. long \times 1 yd. wide? The area of 10 strips must be 10×18 sq. yd., or 180 sq. yd.

RULE. The area of a rectangle is equal to the product of its base and altitude, expressed in like units.

Parallelograms

The four-sided plane figure $ABCD$ is a **parallelogram**, because AB and DC are parallel and AD and BC are parallel.

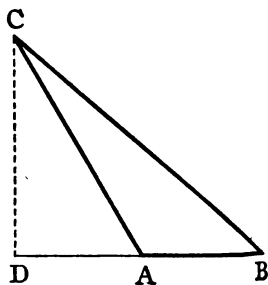
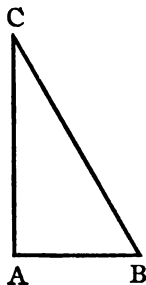
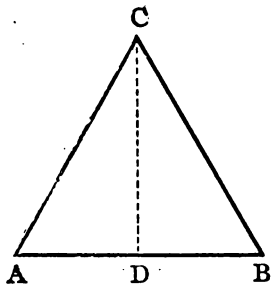


AB (or DC) is the **base** of the parallelogram; EF or CX is the **height** or **altitude** of the parallelogram.

RULE. The area of a parallelogram is equal to the product of its base and altitude, expressed in like units.

e.g. if the base AB is 5 ft. and the altitude EF is 4 ft., the area of the parallelogram is 20 sq. ft.; if the base is 8 ft. and the altitude 9 in., the area is 6 sq. ft.

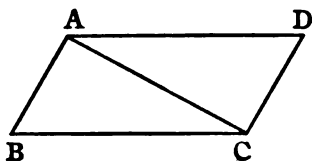
Triangles



These plane figures are called **triangles**, because each of them is bounded by three straight lines.

AB is called the **base**. The height of the triangle is called the **altitude**. It is measured by a perpendicular line from C to AB . In some cases it is necessary to prolong the base, AB , as in the third triangle on page 122.

The triangle ADC equals one half of the parallelogram $ABCD$; therefore:



RULE. The area of a triangle is equal to half the product of its base and altitude, expressed in like units.

e.g. if the base is 4 ft. and the altitude 2 ft., the area is 4 sq. ft.

ORAL EXERCISES

Find the areas of the following rectangles:

	LENGTH	WIDTH		LENGTH	WIDTH
1.	6 ft.	$4\frac{1}{2}$ ft.	6.	18 yd.	20 yd.
2.	20 yd.	7 yd.	7.	250 ft.	40 ft.
3.	30 ft.	15 ft.	8.	80 yd.	80 yd.
4.	7 ft.	6 in.	9.	80 yd.	30 ft.
5.	7 ft. 6 in.	2 ft.	10.	$16\frac{1}{2}$ yd.	10 yd.

WRITTEN EXERCISES

Find the areas of the following rectangles:

	WIDTH	LENGTH
11.	125 yd.	$56\frac{1}{4}$ yd.
12.	80 ft.	1 ft. 9 in.
13.	17 yd.	17 yd.
14.	32 ft.	4 ft. 9 in.
15.	6 ft. 6 in.	5 ft. 6 in.
16.	280 ft. 30 in.	30 in.
17.	75 yd.	42 ft.
18.	600 ft.	2 ft. 9 in.
19.	80 yd.	1 yd. 1 ft.
20.	48 ft.	8 ft. 2 in.

Find the areas of rectangles having the following dimensions:

- | | |
|----------------------|--|
| 1. 27 ft. by 14 ft. | 6. 18 ft. 6 in. by 36 ft. |
| 2. 30 yd. by 27 yd. | 7. $10\frac{1}{2}$ yd. by $6\frac{2}{3}$ yd. |
| 3. 15 rd. by 28 rd. | 8. 75 ft. by 125 ft. |
| 4. 25 ft. by 100 ft. | 9. $62\frac{1}{2}$ rd. by 120 rd. |
| 5. 72 yd. by 140 yd. | 10. 70 yd. by $36\frac{1}{4}$ yd. |

Find the areas of parallelograms having the following dimensions:

- | | |
|----------------------------|---------------------------|
| 1. 72 yd. by 60 yd. | 4. 80 ft. square. |
| 2. 18 ft. 9 in. by 30 ft. | 5. 40 rd. by 117 rd. |
| 3. 25 ft. 6 in. by 100 ft. | 6. 36 yd. by 27 yd. 1 ft. |

7. 16 ft. 8 in. by 40 ft. 9. $18\frac{3}{4}$ ft. by $5\frac{1}{2}$ ft.
8. 720 yd. by 130 yd. 10. 700 ft. by $174\frac{1}{2}$ ft.

Find the areas of triangles having the following dimensions :

- | | |
|-----------------------|--------------------------------|
| 1. Base, 40 ft. | Altitude, 36 ft. |
| 2. Base, 17 in. | Altitude, 20 in. |
| 3. Base, 30 yd. | Altitude, 18 yd. |
| 4. Base, 200 rd. | Altitude, 175 rd. |
| 5. Base, 36 ft. | Altitude, 30 ft. |
| 6. Base, 9 ft. 4 in. | Altitude, 12 ft. |
| 7. Base, 700 yd. | Altitude, 648 yd. |
| 8. Base, 240 rd. | Altitude, 225 rd. |
| 9. Base, 17 yd. 2 ft. | Altitude, 15 yd. |
| 10. Base, 400 ft. | Altitude, $272\frac{1}{2}$ ft. |

Carpeting, Tiling, Plastering, Papering, Etc.

FOR READING

According to rule, when a surface is to be covered with carpet, oilcloth, tiles, plaster, paper, or other material, the entire area of the surface is found, and then is divided by the area of one unit of the covering material. In actual practice, however, this method of calculation is unsatisfactory, because it does not take into account the loss of material in matching patterns, in turning corners, and in using

occasionally an entire strip of carpet or oilcloth or wall paper to cover only an inch or two left uncovered by the last full strip required. Often an entire roll of paper must be paid for, though only a yard or two may have been required. For these and similar reasons, carpet is charged for by strips, wall paper by rolls, oilcloth by the square yard, tiles by the dozen, and plaster by estimate, generally calculated on the approximate number of square yards.

Carpeting

86. How much will it cost to carpet a room 18 ft. \times 12 ft., with carpet 27 in. wide, at \$1.10 per yard, allowing a loss of 3 in. on each strip for matching the pattern, and a charge of 10¢ a yard for laying?

Process

Width of room 18 ft.

Width of strip 27 in. or $2\frac{1}{4}$ ft.

$18 \div 2\frac{1}{4}$ or $\frac{4}{9}$ of 18 = 8, number of strips.

Length of each strip, 12 ft.

+ Loss on each strip 3 in. or $\frac{1}{4}$ ft.

$12\frac{1}{4}$ ft. length of each strip.

$(8 \times 12\frac{1}{4})$ = no. of feet of carpet required,

$(8 \times 12\frac{1}{4}) \div 3$ = no. of yards of carpet required.

$$\frac{49}{4} \times \frac{2}{8} \times \frac{1}{3} = \frac{98}{3} = 32\frac{2}{3} \text{ yd., in commerce } 32\frac{3}{4} \text{ yd.}$$

\$1.10

32 $\frac{3}{4}$

\$36.03 cost of carpet.

3.28 cost of laying.

\$39.31 entire cost.

Explanation. First reckon how many strips 27 in. wide will be required to cover the width of the room, 18 ft.

18 ft. \div 2 $\frac{1}{4}$ ft. (27 in.) = $18 \times \frac{4}{9} = 8$ strips. The loss on each

strip is 3 in. Each strip must then be 12 ft. 3 in. long, or 12 $\frac{1}{4}$ ft. One strip is

12 $\frac{1}{4}$ ft. long, 8 strips

will be 98 ft., or 32 $\frac{2}{3}$

yd. long. (Carpet is

sold only in yards,

or in halves, quarters,

or eighths, so $\frac{2}{3}$ of a

yard will be charged

at the rate of the

nearest fraction of

commerce, in this

case, $\frac{2}{3}$.) The quantity of carpet needed is 32 $\frac{3}{4}$ yd.

The bill will read as follows:

32 $\frac{3}{4}$ yd. of carpet @ \$1.10 \$36.03

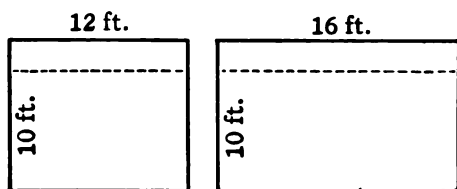
32 $\frac{3}{4}$ yd. laying 3.28

\$39.31

Papering

87. What will be the cost of the paper to cover the walls of a room 16 ft. long, 12 ft. wide, placing

paper to a height of 10 ft.? The paper used is 18 in. wide, and is sold in 6 yd. rolls at 50 ct. per roll. Make a deduction of 20 yd. of paper for doors, windows, etc.



Process

2 walls, 16 ft. = 32 ft.

2 walls, 12 ft. = 24 ft.

56 ft. in width.

56 ft. + 18 in., or $1\frac{1}{2}$ ft. = $\frac{2}{3}$ of 56 = $37\frac{1}{3}$, (in trade)
38 strips.

10 ft. length of 1 strip.

38

(380 ft. + 3) = yards of paper required.

$126\frac{2}{3}$ yd. = 127 yd. (in trade)

less 20 yd.

6 107 yd.

$17\frac{5}{8}$ or 18 rolls.

18 rolls @ $\$ \frac{1}{2}$ = \$9, cost of paper.

Explanation. The room has four rectangular walls: 2 are 16 ft. \times 10 ft.; the other 2 are 12 ft. \times 10 ft. The entire width to be covered with paper is $(16 \times 2) + (12 \times 2) = 32 + 24 = 56$ ft. Each strip is 18 in. or $1\frac{1}{2}$ ft. wide; to cover 56 ft. will require as many strips as $1\frac{1}{2}$ ft. is con-

tained times in 56 ft. $(56 \times \frac{2}{3}) = 37\frac{1}{3}$ strips. To cover $\frac{1}{3}$ of a strip, a whole strip will be used, so we must reckon 38 strips. Each strip is 10 ft. long; 38 strips will be 38×10 ft. = 380 ft. long, changed to yards, $\frac{380}{3} = 126\frac{2}{3}$ or 127 yd. Deducting 20 yd. of paper for doors, windows, etc., leaves 107 yd. to be provided. There are 6 yd. in a roll; to cover 107 yd. will require $\frac{107}{6}$ rolls, or $17\frac{5}{6}$, or 18 rolls, which at 50 ct. a roll, cost \$ 9.

WRITTEN EXERCISE

Find the cost of :

MATERIAL	LENGTH	WIDTH	HEIGHT	LOSS IN MATCHING	SAVING BY DOORS, WINDOWS, ETC.	SIZE PER UNIT	COST PER UNIT
1. Carpet	18 ft.	15 ft.		5 in. to the strip		30 in. wide	\$ 1.25 per yd.
2. Oil cloth	15 ft.	10 ft. 6 in.	ft.	none		1 yd. wide	\$.80 per yd.
3. Paper	18 ft.	15 ft.	8		16 yd.	24 in. wide 6 yd. to a roll	\$.40 per roll
4. Tiles	30 ft.	2 ft. 6 in.	ft.	none		6 in. sq.	\$ 1.50 per doz.
5. Plaster	24 ft.	18 ft.	10	none	75 sq. ft.	1 sq. yd.	\$.30 per sq. yd.
6. Wood flooring	24 ft.	18 ft.		none		boards 3 in. wide, 12 ft. long	\$ 15 per C.

VOLUME OR SOLID CONTENTS

88. A rectangular solid is a solid bounded by six rectangular surfaces.

When the six surfaces are squares, the solid is called a cube.

Solids have three dimensions, length, breadth, and thickness or height.

EXERCISE

1. Cut out of paper or cardboard, 6 squares, each side of the squares being 3 in. Paste the squares together to form a solid. What kind of solid is formed? What are its dimensions?

2. Cut out 6 squares each 4 in. long. Paste the squares together to form a solid. What kind of solid is formed? What are its dimensions?

3. Cut out two rectangles 5 in. long and 4 in. wide; two rectangles, 5 in. long and 2 in. wide; two rectangles 4 in. long and 2 in. wide. Paste to form a solid. What kind of solid is formed? What are its dimensions?

4. Make, by pasting squares, a cube, each dimension being 1 inch. What is its volume?

A cube 1 in. by 1 in. by 1 in. is called an "inch-cube."

5. Make a cube 1 ft. long. What is its volume?

To Find the Volume of a Rectangular Solid

Find the volume of a rectangular solid 5 inches long, 4 inches wide, and 3 inches high.

Process

$$5 \times 4 \times 3 = 60 \text{ cu. in.}$$

Explanation. By using inch-cubes, construct a solid 5 in. long, 4 in. wide, and 3 in. high. The solid can be divided into 3 solids, each 5 in. long, 4 in. wide, and 1 in. high. The volume of the new solid is 20 cu. in. Therefore the volume of the 3 solids, or of the large solid, is 60 cu. in.

RULE. The volume of a rectangular solid is found by multiplying together the length, breadth, and thickness or height, the three dimensions being expressed in like units.

ORAL EXERCISES

Find the solid contents of the following rectangular bins or boxes:

LENGTH	WIDTH	HEIGHT	LENGTH	WIDTH	HEIGHT
	1.			4.	
20 in.	8 in.	6 in.	6 ft.	$8\frac{1}{2}$ ft.	5 ft.
	2.			5.	
25 ft.	10 ft.	2 ft. 6 in.	16 ft.	10 ft.	12 in.
	3.			6.	
40 ft.	25 ft.	9 in.	24 yd.	15 ft.	3 ft.

WRITTEN EXERCISES

Find the solid contents of the following bins or cisterns:

7. 22 ft. \times 8 ft. \times 5 ft.
8. 14 ft. \times $7\frac{1}{2}$ ft. \times 8 ft.
9. 15 ft. \times 10 ft. \times 6 ft. 6 in.
10. 12 ft. 6 in. \times 7 ft. 6 in. \times 11 ft.

89. To Find the Number of Gallons in a Cistern

What is the capacity in gallons of a cistern 8 ft. \times 3 ft. by 11 ft.?

Process

$$\frac{1}{231} \text{ of } (8 \times 3 \times 11) \times 1728 = \frac{13824}{231} = 1974\frac{6}{7} \text{ gal.}$$

Explanation. One liquid gallon contains 231 cu. in. Find the capacity (volume or solid contents) of the cistern. The number of gallons the cistern holds is $\frac{1}{231}$ of the number of cubic inches in the cistern.

The contents are $8 \times 3 \times 11 = 264$ cu. ft. Change to cubic inches. 264×1728 cu. in. $= 456192$ cu. in. $\frac{1}{231}$ of $456192 = 1974\frac{6}{7}$ gal.

To Find the Number of Bushels in a Bin

Find the number of bushels in a bin 5 ft. long, 4 ft. wide, and 3 ft. high or deep.

Process

$$\frac{1}{2150} \text{ of } 5 \times 4 \times 3 \times 1728 = 48 \frac{48}{215} \text{ bu. } \textit{Ans.}$$

$$\begin{array}{r} 2150 \\ 430 \\ \hline 215 \end{array}$$

Explanation. One bushel contains 2150 cu. in. Therefore the number of bushels is $\frac{1}{2150}$ of the number of cubic inches.

Short Process

$$\frac{4}{5} \times 5 \times 4 \times 3 = 48 \text{ bu. } \textit{Ans.}$$

Explanation. 2150 cu. in. is approximately $\frac{5}{8}$ cu. ft. Each bushel occupies about $\frac{5}{8}$ cu. ft. Therefore the number of bushels is $\frac{8}{5}$ of the number of cubic feet.

WRITTEN EXERCISES

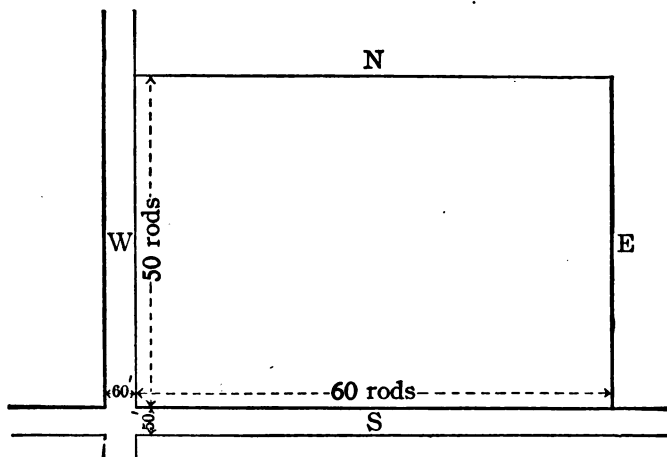
1. Find the number of liquid gallons that can be stored in the vessels described in Examples 1, 4, 7, 10, on pages 130 and 131.
2. Find the approximate number of bushels that can be stored in the vessels described in Examples 2, 5, 8, 12, on pages 130 and 131.

90.

MISCELLANEOUS PROBLEMS

1. How many gallons will a vat hold that is 5 ft. square and 18 in. deep?
2. What is the cost of digging a cellar 30 ft. by 20 ft. and 8 ft. deep, at 75 ct. a cubic yard?
3. Find the cost of carting the excavated earth at 25 ct. a cartload, each cart holding $1\frac{1}{2}$ cu. yd.

4. A silo is 8 ft. square and 20 ft. high; how many cubic yards of ensilage will it contain?



5. Draw to a larger scale the sketch of a plot of land given on this page. Notice the roads on the west and south sides of the plot, and the dimensions of the plot. Make the best possible division into lots. The lots are to be at least 25 ft. wide and about 100 ft. deep. Remember that every lot must have frontage on a street or road and that the streets or roads must be at least 50 ft. wide. The streets running north and south should be about 200 ft. apart; those running east and west should be not more than 400 ft. apart. That plan is the best that provides the greater number of lots of the proper size. (*For home study.*)

6. The weight of ice is .92 of the weight of water. What is the weight of a load of ice 20 ft. long, 9 ft. wide, and 5 ft. high?

7. How many yards of Brussels carpet $\frac{3}{4}$ of a yard wide are required to carpet a room 21 ft. long and 15 ft. wide, if the strips run lengthwise and there is a waste on each strip of 6 in. in matching the pattern?

8. A garden of rectangular shape contains $21\frac{1}{2}$ A. One side is 35 rd. long. How long is the other side?

9. How many cords of wood in a pile 27 ft. long, 12 ft. high, and 8 ft. wide?

FOREIGN MONEY

See Page 100.

FOR READING

91. All civilized nations except Great Britain now use a decimal system of money. The unit is always divided into one hundred parts.

The three countries with which the United States carries on the largest import trade are Great Britain, Germany, and France. Some knowledge of their money values is necessary.

The unit in English money is the pound sterling (£). It is divided into 20 shillings (s.), each shilling being divided into 12 pence (d.),* each penny, into 4 farthings (far.).

The unit in German money is the mark (mk.). It is divided into pfennigs. 100 pf. make 1 mk.

The unit in French money is the franc (fr.). 100 centimes (c.) make 1 fr.

The U. S. value of £1 is \$4.8665 or about \$4.87.

The U. S. value of 1 mk. is \$.238.

The U. S. value of 1 fr. is \$.193.

In reducing U. S. money to English money, or English money to U. S. money, these values must

* The (d.) is the abbreviation of *denarius*, which was a Latin coin of small value.

be used; but for approximate values, we reckon £1 at \$5; 1s. at 25 ct. or 4 to a dollar; 1 mk. at 25 ct. or 4 to a dollar; and 1 fr. at 20 ct. or 5 to a dollar.

ORAL EXERCISE

Find the approximate value in U. S. money of

- | | | |
|-----------|--------------|------------------|
| 1. £ 3½ | 7. £ 10 10s. | 13. 10s. 6d. |
| 2. 50 mk. | 8. 500 fr. | 14. 7 mk. 50 pf. |
| 3. 60 fr. | 9. 800 mk. | 15. 12 fr. 25 c. |
| 4. 50 pf. | 10. 15s. | 16. £ 200 |
| 5. 50 c. | 11. 80 pf. | 17. 200 mk. |
| 6. 50s. | 12. 80 c. | 18. 200 fr. |

ORAL EXERCISE

Find the approximate value in

ENGLISH MONEY	FRENCH MONEY	GERMAN MONEY
1. \$ 25	6. \$ 150	11. \$ 150
2. \$ 100	7. \$ 4.50	12. \$ 4.50
3. \$ 17.50	8. \$ 2.25	13. \$ 2.25
4. \$ 2000	9. \$ 2000	14. \$ 2000
5. \$ 20,000	10. \$ 72	15. \$ 40.25

WRITTEN EXERCISE

Find the exact value of the following in U. S. money:

- | | |
|------------|------------|
| 1. £ 18 | 3. 250 fr. |
| 2. 250 mk. | 4. 16s. |

- | | |
|--------------------|-----------------|
| 5. 90 pf. | 8. 2160 fr. |
| 6. £12 10s.* | 9. 18 fr. 50 c. |
| 7. 160 mk. 50 pf.* | 10. £6 10s. |

WRITTEN PROBLEMS

Use approximate values.

92. 1. At a German hotel, my board bill amounted to 76 mk. 60 pf. What is the value of that bill in U. S. money?

2. In Paris, I paid a bill of 116 fr. How much U. S. money has the same value?

3. In Paris, I wanted to buy a pair of opera glasses. I expected to spend \$12 for them. I paid 55 fr. 50 c. for those I bought. How many fr. less than my estimate did I pay?

4. In London, I bought a rubber coat for £2 10s. I gave a \$20 bill in payment. How much English money did I receive in change?

5. The president of Columbia University receives an annual salary of \$25,000. What is its equivalent in English money? In French money? In German money?

WRITTEN EXERCISES

93. Change 17 fr. 85 c. to U. S. money.

What part of a franc is 85 c.? (Express this decimally.)
Therefore, 17 fr. 85 c. = 17.85 fr.

1 fr. = \$.193 in U. S. money.

17.85 fr. = $17.85 \times \$.193 = \3.44505 or **\$3.45.**

* 10s. = £½; 50 pf. = ½ mark.

Changing francs and marks to U. S. money is simple, because centimes and pfennigs are so easily expressed decimally. Shillings and pence are not so easily expressed, but must first be reduced to the decimal of a pound.

See (c) page 102.

Change £8 12s. 6d. to U. S. money.

Process		Explanation.
12 6.0d.	\$4.87	12s. 6d = .625 of a pound. The question now becomes, What is the value in U. S. money of £8.625? £1 = \$4.87. £8.625 = 8.625 × \$4.87, or \$42.00375. Ans.
20 1.2x5s.	8.625	
£8.625		

\$4.8665 is the real value of one pound sterling, but for school purposes \$4.87 is sufficiently accurate. Banks vary the value of a pound according to money rates of exchange. It fluctuates between \$4.84 and \$4.92. Occasionally it goes beyond even these figures.

EXERCISES

Give U. S. equivalents of:

- | | |
|------------------|--------------------|
| 1. £25 9s. 8d. | 9. 260 mk. 80 pf. |
| 2. £2056 | 10. 1164 fr. 40 c. |
| 3. £70 8s. 4d. | 11. 11,000 mk. |
| 4. £117 7s. 6d. | 12. 24,780 fr. |
| 5. £575 | 13. 462 mk. 60 pf. |
| 6. 19s. 6d. | 14. 5020 mk. |
| 7. £70 18s. | 15. 2175 fr. 35 c. |
| 8. £625 18s. 9d. | 16. 500,000 fr. |

94. Changing U. S. money to foreign money is the reverse process of changing foreign money to U. S. money.

(A) Change \$25 to francs.

What is the U. S. value of 1 franc?

How many francs are there in \$1.93? In \$3.86?
In \$19.30? In \$38.60? In \$193? In \$386?

Process

$$\begin{array}{r}
 129.53^+ \\
 \$.193 \overline{) \$25,000.00} \\
 \underline{193} \\
 570 \\
 \underline{386} \\
 1840 \\
 \underline{1737} \\
 1030 \\
 \underline{965} \\
 650 \\
 \underline{579} \\
 71
 \end{array}$$

Explanation. There are \$.193 in one franc. In \$25 there are as many francs as \$25 contains \$.193, which is $129\frac{53}{100}^+$ times, or 129.53^+ fr. or 129 fr. 53 c.

(B) Change \$25 to marks.

Solution and explanation are exactly the same, using \$.238 for the divisor. \$.238 is the U. S. value of one mark.

(C) Change \$25 to English money.

The solution and explanation are exactly the same as in A except that \$4.87 is used as a divisor. The quotient will be pounds and a decimal of a pound.

$$\begin{array}{r}
 5.13 \\
 \$4,87 \overline{) \$25,00.00} \quad \text{\textit{£}.13 = 2s. 7d.} \\
 \underline{24 \ 35} \\
 65 \ 0 \\
 \underline{48 \ 7} \\
 16 \ 30 \\
 \underline{14 \ 61} \\
 1 \ 69 \quad \text{\textit{£}5 2s. 7d. Ans.}
 \end{array}$$

WRITTEN EXERCISES

Change to exact German money value; to French money.

- | | |
|------------|--------------|
| 1. \$238 | 5. \$450 |
| 2. \$46.85 | 6. \$18.63 |
| 3. \$2000 | 7. \$980 |
| 4. \$75.50 | 8. \$2040.75 |

Change to exact equivalent in English money :

- | | |
|----------------|--------------|
| 9. \$75. | 13. \$4000. |
| 10. \$18.96. | 14. \$11.96. |
| 11. \$2467.50. | 15. \$250. |
| 12. \$36. | 16. \$7.50. |

BILLS

95. 1. Andrew Wheeler bought of Stuart Knox
 8 sewing machines at \$37.50 each.
 1800 needles at $7\frac{1}{2}$ ct. per dozen.
 16 sets attachments at \$2.25 each.
 Make out and receipt the bill.

2. What is the approximate value of the bill in German money? In French? In English?

3. What is the exact value of the bill in English money?

4. In French money?

5. In German money?

6. Make out a bill of purchase in a London harness shop. Use three items. Find total in English money.

7. Give the U. S. value of the entire purchase.

8. Make out a bill of purchase in a French jewelry shop. Use four items.

Find the total cost in French money.

9. Give the U. S. value of the entire purchase.

10. Make out a bill of purchase in a German dry goods store. Use four items. Find the total cost in German money.

11. Give the U. S. value of the entire purchase.

PERCENTAGE

FOR READING

96. Per cent means hundredths. *Per centum* is a Latin phrase meaning *by the hundred*, or hundredths.

The per cent sign is %.

There are several subdivisions of the subject of percentage. Those in common use are:

* Profit and Loss. * Trade Discount. * Commission or Brokerage. Taxes. Duties. * Insurance. * Interest. * Banking.

The underlying principle is the same in all these branches of business arithmetic; the real difference lies only in terms. This book will deal only with practical applications of the subjects marked with an asterisk.

THE RELATION OF PER CENT TO COMMON FRACTIONS AND DECIMAL FRACTIONS

97. Find $\frac{1}{4}$ of 20.

Find .25 of 20.

Find 25 % of 20.

These three examples are different forms of the same problem.

$$\frac{1}{2} \text{ of } 20 = 5$$

$$\frac{1}{25} \text{ of } 20 = \frac{4}{5}$$

$$25\% \text{ of } 20 = 5$$

The process and explanation are exactly the same in the three items: the difference is in the name of the part to be taken.

EXERCISES

Supply the missing number: fraction, decimal fraction, or per cent in each line.

COMMON FRACTIONS	DECIMAL FRACTIONS	PER CENT
1. $\frac{3}{4}$ of 75.	.75 of 75.	75% of 75.
2. $\frac{1}{2}$ of 36.	? of 36.	?% of 36.
3. ? of 60.	.40 of 60.	?% of 60.
4. ? of 48.	? of 48.	33 $\frac{1}{3}$ % of 48.
5. $\frac{1}{8}$ of 56.	? of 56.	?% of 56.
6. $\frac{7}{25}$ of 150.	? of 150.	?% of 150.
7. ? of 72.	? of 72.	37 $\frac{1}{2}$ % of 72.
8. $\frac{1}{5}$ of 35.	? of 35.	?% of 35.
9. $\frac{7}{9}$ of 63.	? of 63.	?% of 63.
10. $\frac{2}{3}$ of 90.	? of 90.	?% of 90.
11. $\frac{2}{25}$ of 100.	? of 100.	?% of 100.
12. $\frac{2}{3}$ of 96.	? of 96.	?% of 96.

EXERCISES

98. 1. Change to equivalent decimal fractions;
to equivalent common fractions:

70%; 30%; 62 $\frac{1}{2}$ %; 9%; 4 $\frac{1}{2}$ %; $\frac{3}{4}$ %; $\frac{1}{2}$ %.

2. Change to equivalent decimal fractions; to equivalent rates per cent:

$$\frac{5}{6}; \frac{3}{50}; \frac{3}{200}; \frac{5}{16}; \frac{1}{300}; \frac{7}{80}; \frac{5}{12}; \frac{7}{500}; \frac{3}{80}; \frac{1}{30}.$$

3. Change to equivalent whole or mixed numbers:

$$200\%; 150\%; 725\%; 100\%; 325\%; 133\frac{1}{3}\%; 250\%; 1000\%.$$

99. Memorize this table; it contains the most important per cents, and their common fraction equivalents.

100 % = 1	12½ % = $\frac{1}{8}$	10 % = $\frac{1}{10}$	33⅓ % = $\frac{1}{3}$
50 % = $\frac{1}{2}$	37½ % = $\frac{3}{8}$	20 % = $\frac{1}{5}$	66⅔ % = $\frac{2}{3}$
25 % = $\frac{1}{4}$	62½ % = $\frac{5}{8}$	30 % = $\frac{3}{10}$	133⅓ % = $\frac{4}{3}$
75 % = $\frac{3}{4}$	87½ % = $\frac{7}{8}$	40 % = $\frac{2}{5}$	16⅔ % = $\frac{1}{6}$
200 % = 2	112½ % = $\frac{9}{8}$	60 % = $\frac{3}{5}$	83⅓ % = $\frac{5}{6}$
150 % = $\frac{3}{2}$	137½ % = $\frac{11}{8}$	70 % = $\frac{7}{10}$	8⅓ % = $\frac{1}{12}$
125 % = $\frac{5}{4}$	162½ % = $\frac{13}{8}$	80 % = $\frac{4}{5}$	41⅔ % = $\frac{5}{12}$
175 % = $\frac{7}{4}$	187½ % = $\frac{15}{8}$	90 % = $\frac{9}{10}$	58⅓ % = $\frac{7}{12}$

100. This table is given for reference only.

2 % = $\frac{1}{50}$	7 % = $\frac{7}{100}$	$\frac{1}{2}$ % = $\frac{1}{200}$	$\frac{3}{4}$ % = $\frac{3}{400}$
4 % = $\frac{1}{25}$	6¼ % = $\frac{1}{16}$	1½ % = $\frac{3}{200}$	3⅓ % = $\frac{1}{30}$
1 % = $\frac{1}{100}$	6⅔ % = $\frac{1}{15}$	7½ % = $\frac{3}{40}$	$\frac{1}{3}$ % = $\frac{1}{300}$

TERMS USED IN PERCENTAGE

101. The terms used in percentage are BASE, RATE, PERCENTAGE, and AMOUNT or DIFFERENCE.

The base is the number of which some part or some per cent is to be found.

Find 18 % of 50. 50 is the **base**.

The **rate**, or the **rate per cent**, is the number of hundredths to be found.

Find 18 % of 50. 18 % or .18 is the **rate**.

The **percentage** is the result obtained by finding a given per cent of the base.

Find 18 % of 50. $.18 \text{ of } 50 = 9$. 9 is the **percentage**.

The **amount** is the base plus the percentage.

Find the amount of 50 increased by 18 % of itself.

$18 \% \text{ of } 50 = 9$. $50 + 9 = 59$. 59 is the **AMOUNT**.

The **difference** is the base minus the percentage.

Find the difference after 50 has been decreased by 18 % of itself.

$18 \% \text{ of } 50 = 9$. $50 - 9 = 41$. 41 is the **DIFFERENCE**.

PROCESSES IN PERCENTAGE, WITH THE CORRESPONDING PROCESSES IN FRACTIONS

I. IMPORTANT PROCESSES IN FRACTIONS

102. The three chief processes in fractions are :

1. To find a fractional part of a number.
2. To find what fractional part one number is of another.

3. To find a number when a fractional part of it is known. (See Art. 39 for the three cases of this type.)

1. Find $\frac{3}{8}$ of 32.

Process

$$\frac{3}{8} \text{ of } 32 = 12 \text{ Ans.}$$

2. 12 is what part of 32 ?

Process

$$12 = \frac{12}{32} \text{ of } 32 = \frac{3}{8} \text{ Ans.}$$

3. (a) 12 is $\frac{3}{8}$ of what number ?

Process

$$\frac{3}{8} \text{ of the number} = 12.$$

$$\frac{1}{8} \text{ of the number} = (\frac{1}{3} \text{ of } 12), \text{ or } 4.$$

$$\frac{3}{8} \text{ of the number} = 32 \text{ Ans.}$$

3. (b) 44 is $\frac{3}{8}$ more than what number ?

Process

$44 + \frac{3}{8}$ of the number $= \frac{11}{8}$ of the original number.

$\frac{11}{8}$ of the number $= 44$.

$\frac{1}{8}$ of the number $= (\frac{1}{11} \text{ of } 44)$, or 4.

$\frac{3}{8}$ of the number $= 32$ *Ans.*

(c) 20 is $\frac{3}{8}$ less than what number?

Process

$20 = \frac{5}{8}$ of the original number.

$\frac{5}{8}$ of the number $= 20$.

$\frac{1}{8}$ of the number $= (\frac{1}{5} \text{ of } 20)$, or 4.

$\frac{3}{8}$ of the number $= 32$ *Ans.*

II. IMPORTANT PROCESSES IN PERCENTAGE

103. The three chief processes in percentage are the same as those in fractions.

1. To find a per cent of a number.
2. To find what per cent one number is of another.
3. To find a number when a per cent of that number is known.

TO FIND THE PERCENTAGE, THE BASE AND RATE BEING GIVEN

104. 1. To find a per cent of a number.

(A) When the rate per cent can easily be reduced to an equivalent common fraction or mixed number: as 25 %, $33\frac{1}{3}$ %, 10 %, etc., all easily reducible to simple common fraction form; or 150 %, 125 %, etc., all easily reducible to mixed numbers.

225 %, 400 %, etc., all easily reducible to simple whole or mixed numbers.

B. When the rate per cent must be used as a decimal: as 17 %, $31\frac{1}{2}$ %, $\frac{1}{2}$ %, * $\frac{7}{8}$ %, etc.

ORAL EXERCISES

105. 1. Find 25 % of 60; of 48; of 72; of 400; of 1000.

2. Find 50 % of 72 bu.; of 96 cows; of 150 books; of \$ 640; of 1200 soldiers.

3. Find 75 % of 12; of 80; of 800; of 4000; of 20,000.

4. Find $12\frac{1}{2}$ % of \$ 16; of \$ 40; of 80 ct.; of 96 ct.; of 400 yd.

5. Find $66\frac{2}{3}$ % of 9; of 60; of 75; of 150; of 3000.

6. Find $33\frac{1}{3}$ % of 12 mi.; of 144 sq. in.; of 1760 yd.; of 5280 ft.; of 1728 cu. in.

7. Find $37\frac{1}{2}$ % of 24; of 40; of 72; of 800; of 4000.

8. Find $62\frac{1}{2}$ % of 16 oz.; of \$ 400; of \$ 1.60; of \$ 2400; of 16,000 bricks.

9. Find $87\frac{1}{2}$ % of the numbers used in Examples 7 and 8.

10. Find 20 %, 40 %, 60 %, and 80 % of 200; of 3000; 4500; 5000; 10,000.

* At times it may be simpler to use $\frac{1}{800}$ or $\frac{1}{1600}$ than .00 $\frac{1}{2}$ or .00 $\frac{1}{4}$. The pupil must use his judgment in deciding which form is easier.

ORAL PROBLEMS

106. 1. A school which had 720 pupils, now has 125% as many. How many pupils are in the school now?

2. Find the number of feet (5280 ft. to the mile) in $133\frac{1}{3}\%$ of a mile; 150% of a mile; 125% of a mile; 120% of a mile; 200% of a mile.

3. I sold a boat that cost \$16 for 175% of its cost. How much money did I get for the boat?

4. John has 64 marbles; Henry has $112\frac{1}{2}\%$ of that number. How many marbles has Henry?

5. In 1900 a certain town contained 20,000 inhabitants. Its population, in 1910, was 150% of that number. How many inhabitants were there in 1910?

ORAL EXERCISES

107. 1. Find 2% of 8; of 20; of 60; of 200.

2. Find 6% of \$60; of 600 lb.; of 4000 A.; of \$7500; of 8000 mi.

3. Find 4% of 25; of 80; of 250; of 1600; of 5000.

4. Find 15% of 50; of 200; of 700; of 3000; of 20,000.

5. Find 35% of 40; of 400; of 4000; of 40,000; of 400,000.

6. Find 8% of 200. Use the same base and find 7%; 9%; 21%; 32%; 41%; $6\frac{1}{2}\%$; $4\frac{1}{2}\%$ of it.

ORAL DRILLS

108. Find value of ? :

	BASE	RATE %	PERCENTAGE	AMOUNT	DIFFERENCE
1.	60	25	?		
2.	40	40		?	
3.	200	75	?		
4.	150	$33\frac{1}{3}$?		
5.	72	$12\frac{1}{2}$?
6.	120	60		?	
7.	1000	8	?		
8.	400	25 ?
9.	56	$14\frac{1}{4}$?		
10.	2500	10		?	
11.	700	20	?		
12.	3000	15		?	
13.	10,000	30	?		
14.	6000	20 ?
15.	48	75	?		
16.	160	50		?	
17.	72	100		?	
18.	\$ 12.50	20	?		
19.	\$ 125	80	?		
20.	500	$33\frac{1}{3}$?		

ORAL EXERCISES

109. 1. Find $\frac{1}{2}\%$ of 300; of 400; of 360; of 1000; of 1250.

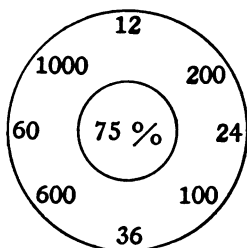
2. Find $\frac{1}{3}\%$, $\frac{2}{3}\%$, $\frac{1}{5}\%$, $\frac{1}{4}\%$, $\frac{1}{6}\%$ of 2400.

3. Find the difference between $\frac{1}{8}\%$ of 1600 and $\frac{3}{8}\%$ of 4800.

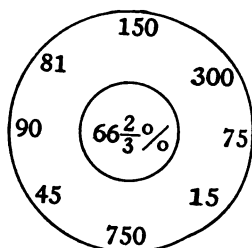
ORAL DRILLS

The Circle

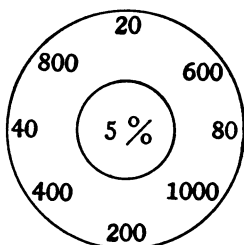
110. The number in the center is the rate. Each number in the ring is the base. Find the percentage.



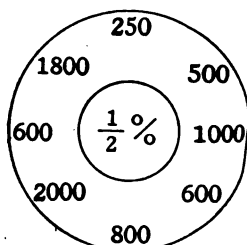
No. 1.



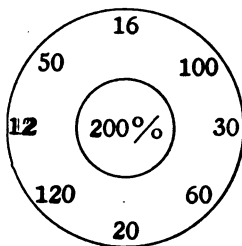
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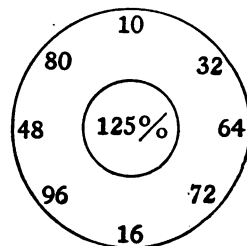
No. 3.



No. 4.



No. 5.



No. 6.

ORAL PROBLEMS

- 111.** 1. How much is $12\frac{1}{2}\%$ of \$40?
2. Find $33\frac{1}{3}\%$ of 210 boys.
3. An article that cost 80 ct. was sold at a gain of 10%. How much was gained?
4. A boy had 70 marbles. After a game of marbles, he had 20% more. How many did he win?
5. The wages of a carpenter were \$24 a week. He received an increase of $16\frac{2}{3}\%$. What is the amount of the increase?
6. How many pounds are there in 20% of a ton of coal? In 40% of a ton?
7. If a yard of cloth costs \$3, how much will $\frac{3}{4}$ of a yard cost? (State the same problem in percentage.)
8. There are two grades of coffee; the better kind costs 40 ct. a pound; the cheaper costs 80% as much. How much does the cheaper coffee cost per pound?
9. One train travels a distance of 200 mi. in 5 hr. Another train goes the same distance in 90% of the time. How long does it take the second train?
10. One bin holds 200 bu. of grain. Another bin holds 20% more. How many bushels does the second bin hold?

11. A plumber's apprentice receives \$1.50 a day. The plumber's wages are 100 % more than the wages of the apprentice. How much does the plumber receive per day ?

12. One suit of clothes was marked \$25. Another suit was marked 40 % higher. What was the price of the second suit ?

13. Ten years ago a town had a population of 1000 people. To-day its population is 16 % greater. What is the population of the town to-day ?

A railroad company advanced the price of its tickets. Find answers for ? in each case.

	ORIGINAL PRICE	INCREASE	AMOUNT OF INCREASE	NEW PRICE
14.	60 ct.	33 $\frac{1}{3}$ %	?	?
15.	96 ct.	16 $\frac{2}{3}$ %	?	?
16.	\$1.20	30%	?	?
17.	2.10	25%	?	?
18.	3.00	20%	?	?

19. A baker uses 400 bbl. of flour in one week ; the next week he uses 30 % more. How many barrels are used during the second week ?

20. 260 bbl. of apples were shipped to a grocer. 5 % of them became rotten. How many barrels were spoiled ?

21. A tailor earned \$15 in one week. The next

week he earned 6% more. How much did he earn the second week?

22. A factory which formerly made 250 pairs of shoes each day increased its product by 8%. How many more pairs of shoes does it make now per day?

23. A boat carried 180 tons of coal. After $33\frac{1}{3}\%$ had been consumed, how many tons were left?

24. A piano was marked \$600. It was sold at a reduction of 15%. How much was paid for it?

25. Last year, the apple trees in an orchard produced 350 barrels of apples. This year they produced $14\frac{2}{7}\%$ more. How many barrels in this year's crop?

WRITTEN PROBLEMS

112. NOTE 1. In all these examples there will be given the **BASE** and the **RATE**, to find the **PERCENTAGE**, or the **AMOUNT** or **DIFFERENCE**.

The general rule is

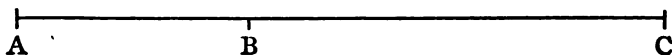
The percentage equals the base multiplied by the rate.

The base plus the percentage equals the amount.

The base minus the percentage equals the difference.

1. How much is 24% of \$1120?

2. The distance between towns A and C is 240 mi. The distance between A and B is 36% as great as the distance from A to C. How far is it from A to B? How far is it from B to C?



3. The height of an office building is 240 ft. A building standing next to it is only 42 % as high. How high is the smaller building?

4. A man weighs 210 lb. His son weighs 35 % as much. Find the son's weight.

5. There are 190 school days in a year. A boy is present 90 % of the total number of days. How many days is he present? How many days is he absent?

6. One man weighs 280 lb. Another weighs 85 % as much. Find the weight of both together.

7. \$12,500 is to be divided between 2 men. One man receives 46 % of this amount. How much money should each man receive?

8. There are 860 chickens in a farmyard. 35 % of them are Rhode Island Reds. The remainder are White Leghorns. How many of each kind are there?

9. An express train travels at the rate of 64 mi. an hour. A local train travels only 85 % as many miles an hour. How many miles will the local train make in 5 hr.?

10. The cost of sending goods by railroad was \$430. The cost of sending the same goods by water is 76 % as much. How much is saved by sending the goods by water?

11. The total vote polled by two candidates for

office was 26,500. One candidate received 44% of the total vote cast. Find the number of votes cast for each candidate.

12. Chairs were marked \$15 each. Later the price was reduced 12%. How much money would be saved by buying 6 chairs at the present price?

13. A wheat field produced 4300 bu. of wheat last year. This year it produced 8% less than last year. How many bushels were produced this year?

14. A peach orchard yielded 960 baskets of peaches last year. This year the crop was 9% greater. How many baskets of peaches were produced this year?

15. By putting in new machinery, a toy factory increased the number of toys made each month by 16%. If it had made an average of 550 toys a month previously, how many does it make now?

16. An express company employs 90 drivers. Formerly it paid them \$15 a week. Recently the men obtained an increase of 18% in their wages. How much more money must the company pay in wages each week than formerly?

17. The population of the city of New York is approximately 4,740,000. $6\frac{2}{3}\%$ of the people have telephones. How many people have telephones?

18. The cost of printing a book was \$275. 36%

of that amount was spent for labor; 48 % for paper. How much was spent for labor? For paper?

19. The Metropolitan Life Insurance building in New York is 700 ft. high. The Singer building is $87\frac{1}{2}$ % as high. How high is the Singer building?

20. A company sells 60,000 lb. of ice. 27 % of it is natural ice; the rest is artificial ice. How many pounds of each kind of ice are sold?

21. A house and lot together cost \$6500. The lot cost 32% of that amount. Find the cost of the lot. Of the house.

22. An apprentice who worked at a machine that made lead pencils spoiled 4 pencils out of every hundred. How many pencils were spoiled out of 650?

23. The railroad fare between two cities is \$55. The fare by boat is 84 % as much. How much will 6 persons save by going by boat?

24. A farmer received \$3500 for his apples, peaches, and pears. 22 % of the amount was received for apples, 45 % for peaches, and the rest for pears. How much money was received for each kind of fruit?

25. 15 arithmetic examples were given in an examination. $73\frac{1}{3}$ % of them were solved correctly. How many examples were not correct?

26. A horse that cost \$650 is sold at a gain of 22 %. Find the selling price of the horse.

27. A house that is worth \$24,500 is insured for 84 % of its value. For how much is it insured?

28. 10 years ago the population of a city was 520,000. Since then the population has increased 24 %. What is the population of the city now?

29. A farmer wanted to raise turkeys and bought 320 young ones. 6 out of every 10 died before they were grown up. How many of the turkeys did the farmer raise?

30. The expenses of governing a city, in one year, were \$125,000. The next year they were 16 % greater. The third year they were 12 % greater than the second year. Find the expenses for the second year; also for the third year.

31. The receipts of a business for one day were \$8250. The next day the sales were only 92 % of the first day. Find the amount of sales for the second day. For both days.

32. A furnace consumed 85 tons of coal a week. A new furnace was put in which consumed only 72 % as much coal per week.

(a) How many tons of coal were saved by the use of the new furnace in 20 weeks?

(b) How much money was saved if coal cost \$6 a ton?

33. How much greater is 83 % of \$ 750 than 72 % of \$ 750 ?

34. How much less is 14 % of \$ 920 than 42 % of \$ 920 ?

35. The length of the Rhine river is 48 % of the length of the St. Lawrence river. The St. Lawrence is 2,000 mi. long ; how long is the Rhine ?

Find values for ? in the following :

		POPULATION 1900	% INCREASE IN 10 YR.	PRESENT POPULATION
36.	Boston .	560,000	18 %	?
37.	Los Angeles	102,000	211 %	?
38.	Louisville .	204,000	9 $\frac{1}{2}$ %	?
39.	Cincinnati .	326,000	11 %	?
40.	New York .	3,437,000	38.7 %	?

41. During the first year a newspaper had a circulation of 95,000 copies. The second year the circulation increased 12 %. The third year it decreased 6 % from the amount of the second year. Find the circulation for the second year. For the third year. For the three years.

42. The total receipts of a shoe store were \$ 1250. Of that amount 42 % was received for men's shoes, 20 % for children's shoes, and the rest for ladies' shoes. How much money was received for men's shoes? For children's shoes? For ladies' shoes?

43. 150 words were given to a class in an examination in spelling. George spelled 96 % of the words correctly. Henry had 84 % correct, and William 60 % correct. How many words did George spell incorrectly? Henry? William?

A piano factory changed the prices of its pianos. Find values for?

	PIANO	FORMER PRICE	INC. %	DEC. %	NEW PRICE
44.	Style A	\$ 325	12 %		?
45.	Style B	350		16 %	?
46.	Style C	500	16 %		?
47.	Style D	650		22 %	?
48.	Style E	800	23 %		?

49. Prizes amounting to \$10,000 were distributed at an aviation meeting. The winner received 56 % of the money; the second, 38 %; the third, the remainder. How much did each receive?

50. The state of Illinois produced the greatest number of bushels of corn. Iowa produced 73 % as much as Illinois; Missouri 56 % as much as Iowa. If the amount of corn produced in Illinois was worth \$1,500,000, what was the value of the corn produced in Iowa? In Missouri?

51. Find the value of the corn produced in the three states.

TO FIND THE RATE, THE BASE AND PERCENT-AGE BEING GIVEN

113. 2. To find what per cent one number is of another.

1. $3 = \text{what } \% \text{ of } 12 ?$

Process

$$3 = \frac{3}{12} \text{ of } 12$$

$$\frac{3}{12} = \frac{1}{4} = 25 \% \text{ Ans.}$$

Explanation

Before we can find what per cent one number is of another, we must find what *part* one number is of another.

3 is what *part* of 12 ?

$$3 = \frac{3}{12} \text{ of } 12.$$

$$\frac{3}{12} = \frac{1}{4}; \frac{1}{4} = 25 \%.$$

Therefore $3 = 25 \% \text{ of } 12.$

2. $3 = \text{what } \% \text{ of } 11 ?$

Process

$$3 = \frac{3}{11} \text{ of } 11$$

$$11 \overline{) 3.00}$$

$$.27 \frac{3}{11} = 27 \frac{3}{11} \% \text{ Ans.}$$

Explanation

$$3 = \frac{3}{11} \text{ of } 11.$$

Change $\frac{3}{11}$ to .00's.

$$\frac{3}{11} = .27 \frac{3}{11}.$$

$$.27 \frac{3}{11} = 27 \frac{3}{11} \% \text{ Ans.}$$

ORAL EXERCISES

114. 1. $7 = \text{what per cent of } 28 ? \text{ of } 35 ? \text{ of } 42 ?$
of $56 ? \text{ of } 14 ? \text{ of } 21 ?$

2. $12 = \text{what per cent of } 36 ? \text{ of } 24 ? \text{ of } 60 ?$
of $96 ? \text{ of } 120 ? \text{ of } 30 ? \text{ of } 48 ? \text{ of } 72 ?$

3. $9 = \text{what per cent of } 45 ? \text{ of } 18 ? \text{ of } 36 ?$
of $72 ? \text{ of } 180 ? \text{ of } 90 ? \text{ of } 54 ? \text{ of } 900 ?$

4. $20 = \text{what per cent of } 60 ? \text{ of } 80 ? \text{ of } 120.$

5. $16 =$ what per cent of 64? of 128? of 32?
of 16? of 160? of 1600? of 320?

6. $25 =$ what per cent of 150? of 250? of 125?
of 75? of 50? of 500? of 400?

7. $10 =$ what per cent of 60? of 80? of 100?
of 30? of 20? of 5? of 2?

8. $24 =$ what per cent of 48? of 72? of 240?
of 480? of 96? of 12? of 6?

9. $150 =$ what per cent of 300? of 450? of 600?
of 1500? of 3000?

10. $2000 =$ what per cent of 4000? of 5000? of
12,000? of 10,000? of 80,000?

ORAL EXERCISES

115. 1. $4 =$ what per cent of 7? of 9? of 11?

2. $5 =$ what per cent of 6? of 7? of 9? of 11?

3. $1 =$ what per cent of 7? of 9? of 11? of 12?

4. $13 =$ what per cent of 2? of 4? of 8? of 10?
of 13? of 20? of 9?

5. $7 =$ what per cent of 3? of 4? of 6? of 7?

ORAL EXERCISES

116. 1. $7\frac{1}{2} =$ what % of 15?

Explanation

Process

$$\frac{7\frac{1}{2}}{15} = \frac{1}{2} \text{ or } 50\% \quad \text{Ans.}$$

Change the complex frac

tion $\frac{7\frac{1}{2}}{15}$ to a simple fraction
 $\frac{1}{2} = 50\% \quad \text{Ans.}$

2. $7\frac{1}{2}$ is what % of 23?

1st Process

$$\frac{7\frac{1}{2}}{23} = \frac{15}{46}$$

$$\frac{15}{46} = .321\frac{4}{23} = 32\frac{14}{23}\% \text{ Ans.}$$

2nd Process

$$\frac{7\frac{1}{2}}{23} = \frac{7.50}{23} \frac{.3214}{7.50}$$

$$\begin{array}{r} 69 \\ 60 \\ 46 \\ 14 \\ 23 \end{array}$$

$$.321\frac{4}{23} = 32\frac{14}{23}\%.$$

1st Explanation

Change the complex fraction $\frac{7\frac{1}{2}}{23}$ to $\frac{15}{46}$ by multiplying both numerator and denominator by 2.

$$\frac{15}{46} = 32\frac{14}{23}\%.$$

2nd Explanation

$\frac{7\frac{1}{2}}{23}$ is a complex fraction; reduce it to a simple fraction by dividing the numerator by the denominator.

3. $1\frac{1}{2}$ = what per cent of 3? of 6? of 10? of 15?

4. $2\frac{1}{4}$ = what per cent of $4\frac{1}{2}$? of $6\frac{3}{4}$? of 9?

5. $2\frac{1}{2}$ = what per cent of 5? of 10? of 15? of 25?

6. 1.25 = what per cent of $2\frac{1}{2}$? of 5? of 3.75?

7. 1.2 = what per cent of 7.2? of 4.8? of 12?

8. $\frac{1}{4}$ = what per cent of $\frac{1}{3}$? of $\frac{2}{3}$? of $\frac{5}{6}$? of $\frac{5}{12}$?

PROBLEMS

117. NOTE 1. In nearly all the following problems there will be given the BASE and the PERCENTAGE to find the RATE.

The general rule is

The rate equals the percentage divided by the base.

NOTE 2. Results may be proved or tested by multiplying the base by the rate; the product should equal the percentage

ORAL PROBLEMS

118. 1. 6 in. is what per cent of a foot? 9 in. is what per cent of a foot?

2. 33 ct. is what per cent of \$1?

3. 33 ct. is what per cent of \$2? Of \$5?

4. 8 in. is what per cent of 2 ft.? Of 4 ft.? Of 5 ft.?

5. The length of a room is 24 ft., the width is 16 ft. What per cent of the length is the width?

6. 6 oz. is what per cent of a pound (avoir.)? Of 3 lb.?

7. A pupil has 5 out of 10 examples correct. What per cent has he correct?

In a "spelling-bee" 5 girls made the following scores. Find values for?

		NO. OF WORDS GIVEN	NO. SPELLED CORRECTLY	%
8.	Mary . .	50	49	?
9.	Jane . .	50	46	?
10.	Kate . .	50	45	?
11.	Agnes .	50	48	?
12.	Florence .	50	50	?

13. Who won the spelling match? Why?

14. The length of a rectangle is 18 ft. Its width is 12 ft. What per cent of the length is the width?

15. There are 60 children in a class. 40 of them are boys. What per cent of the class are girls?

16. A baseball club plays 25 games and wins 21. What per cent of the games does it lose?

17. Five years ago a town had a population of 900 people. To-day its population is 2700. Find the per cent of increase.

18. The height of a church steeple is 150 ft. The height of a flagpole is 100 ft. The height of the flagpole is what per cent of the height of the steeple?

19. A factory employs 200 men and 50 boys. What per cent of the total number employed are boys?

20. What per cent of the total number employed are men?

21. A chair that cost \$21 is sold for \$42. What per cent of the cost is gained on the sale?

22. A soldier fired 40 shots at a target. He hit the target 35 times. What per cent of his chances did he miss?

23. A grocer who bought 300 doz. eggs found that 2 in every 8 were bad. What per cent of the eggs were good?

ORAL PROBLEMS

119. 1. 3 days are what per cent of a week?

2. A girl earns \$11 a week. What per cent

of her salary does she lose if she is absent for 4 days?

3. A man has to travel 150 mi. What per cent of his journey has he completed, when he has traveled 70 mi.?

4. William tried to solve 9 examples. He solved 7 correctly. What was his per cent in arithmetic?

5. A table that cost \$12 was sold at a gain of \$4. Find the gain per cent.

6. Towels that cost \$9 a dozen were sold for \$13 a dozen. Find the gain per cent.

7. An empty bucket weighs 3 lb. When the bucket is filled with water, it weighs 8 lb. What per cent of the total weight is due to the weight of the water?

8. Harry's salary was increased from \$9 to \$15 a week. Find the per cent of increase.

9. Handkerchiefs were bought for \$3 a dozen and sold for 50 ct. each. Find the gain per cent.

10. The price of a quart of milk was increased from 8 to 11 ct. Find the per cent of increase.

ORAL PROBLEMS

120. 1. $3\frac{1}{2}$ lb. are what per cent of 10 lb.?

2. A farmer gives his hired man $2\frac{1}{2}$ bu. of apples for picking 40 bu. What per cent does the hired man receive?

3. A man is expected to work 6 da. a week. If he is absent $1\frac{1}{2}$ da. a week, what per cent of the week does he lose?

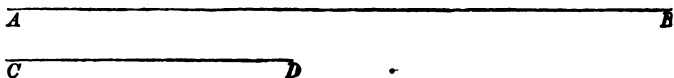
4. What per cent of the week is he at work?

5. An automobile was going at the rate of 20 mi. an hour. The speed was then increased to 27 mi. an hour. By what per cent had the speed been increased?

6. The length of a room is $2\frac{1}{2}$ times the width. What per cent of the width is the length? What per cent of the length is the width?

7. A boy had 60 newspapers. He sold all except 5. What per cent of his papers did he fail to sell?

8. The length of AB is $3\frac{1}{2}$ in. The length of CD is $1\frac{1}{2}$ in. The length of CD is what per cent of the length of AB ?



9. Rice that cost 6 ct. a pound is sold for $8\frac{1}{2}$ ct. a pound. Find the per cent gained.

10. One store charges $10\frac{1}{2}$ ct. a pound for oatmeal. Another store charges 7 ct. a pound for oatmeal. The second price is what per cent of the first price?

WRITTEN PROBLEMS

121. 1. Harry had 48 marbles. He won 16 more. What per cent of his original number did he win?

2. A square foot contains 144 sq. in. What per cent of a square foot are 72 sq. in.? 36 sq. in.?

3. A baseball club plays 36 games. It wins 28 games. What per cent of the games does it win?

4. What per cent of the games does it lose?

5. The total number of pupils in a class is 48. All are present except 3. What per cent of the class is present?

Find answers for ? in each of the following:

		VALUE	REDUCED PRICE	% REDUCTION
6.	Teaspoons . .	\$ 4.50	\$ 3.00	?
7.	Tablespoons .	15.00	12.50	?
8.	Sugar Spoons .	1.50	1.00	?
9.	Knives . . .	12.00	10.00	?
10.	Forks	12.00	9.50	?
11.	Carving Knives	9.00	6.50	?
12.	Berry Spoons .	6.00	4.50	?
13.	Sugar Bowls .	1.50	1.25	?
14.	Cream Bowls .	1.75	1.25	?
15.	Pitchers . . .	2.50	1.75	?

122**WRITTEN PROBLEMS**

1. A company had 127 lots to sell. It sold 55 the first month, and the remainder the second month. What per cent of the lots was sold the first month? The second month?

2. 97 bbl. of apples were shipped by express. 27 bbl. were spoiled. What per cent was spoiled? What per cent was not spoiled?

3. There were 195 school days in a year. Mary was present 185 days. What per cent of the total time was she present?

4. In a school containing 560 pupils, 430 were promoted. What per cent was promoted?

5. A desk that cost \$39 was sold for \$51. Find the per cent gained.

6. An orchard which yielded 275 baskets of peaches last year, produced only 220 baskets this year. Find the per cent of reduction.

7. A wagon carrying a ton of coal (2000 lb.) weighed 2750 lb. What per cent of the total weight was due to the weight of the wagon?

8. A grocer sells 80 lb. of first quality tea, and 70 lb. of second quality tea. What per cent of his total sales is of the first quality?

9. If a pump fills a tank in an hour, what per cent of the tank will be filled in 32 min.?

10. In a town containing 700 people, all except 50 could read and write. What per cent of the people were illiterate?

WRITTEN PROBLEMS

123. 1. $2\frac{1}{2}$ qt. is what per cent of a gallon?
 $5\frac{1}{2}$ qt. is what per cent of a gallon?

2. A wall 22 ft. long is papered with strips of paper $1\frac{1}{2}$ ft. wide. Each strip is what per cent of the length of the wall?

3. $4\frac{1}{2}$ hr. are what per cent of a day? $8\frac{1}{2}$ hr. are what per cent of a day?

4. If a man receives \$18 a week, what per cent of a week must he work to earn \$ $5\frac{1}{2}$?

5. If 580 lb. of flour are divided equally into 29 packages, what per cent of the flour is in each package?

6. $2\frac{1}{2}$ doz. are what per cent of a gross (12 doz.)?

7. A boy who weighed 89 lb. in January weighed 103 lb. in June. His weight in June was what per cent of his weight in January?

8. A silver statue and a wooden base weighed 28 lb. The base weighed $3\frac{1}{2}$ lb. What per cent of the total weight was silver?

9. A race track is $\frac{1}{8}$ of a mile long. What per cent of a mile has a man finished when he has gone $1\frac{1}{8}$ laps?

10. An ocean vessel sailed 610 mi. in a day. What per cent of that distance was sailed in $5\frac{1}{2}$ hr.?

IMMIGRATION

The total immigration into the U. S. during the last three decades was in round numbers.

1881—1890	5,247,000
1891—1900	3,688,000
1901—1910	8,795,000

11. What per cent of the total immigration of the 30 years entered during the first decade?

NOTE. A decade is 10 years.

12. During the second decade?
 13. During the third decade?
 14. What per cent of the immigration in the first decade was that of the second?
 15. What per cent of the immigration in the second decade was that of the third?

TO FIND THE BASE, THE PERCENTAGE AND
RATE BEING GIVEN

124. 3. (a) To find a number when a per cent of that number is given.

First Case

1. 12 is $\frac{1}{8}$ of what number?
 12 is $.12\frac{1}{2}$ of what number?
 12 is $12\frac{1}{2}\%$ of what number?
 2. $12 = 37\frac{1}{2}\%$ of what number?

Process

FIRST METHOD

$37\frac{1}{2}\%$ or $\frac{3}{8}$ of a number = 12

$\therefore \frac{1}{8}$ of the number = $\frac{1}{3}$ of 12 = 4

$\frac{8}{8}$ of the number = 32 *Ans.*

Explanation. $37\frac{1}{2}\%$ of a number or $\frac{3}{8}$ of that number equals 12; therefore $\frac{1}{8}$ of the number equals $\frac{1}{3}$ of 12 or 4; and $\frac{8}{8}$ of the number equals 32 *Ans.*

SECOND METHOD

Shorter Explanation

$$12 \div .37\frac{1}{2} = 12 \div \frac{3}{8}^*$$

$\frac{8}{8}$ of the number equals 12;
the number = $\frac{8}{3}$ of 12 = 32 *Ans.*

or $12 \times \frac{8}{3} = 32$ *Ans.*

ORAL EXERCISES

1. 8 is 25% of what number? 50%? $33\frac{1}{3}\%$?
10%? $12\frac{1}{2}\%$? 20%? $\frac{2}{3}\%$?

2. 18 is $66\frac{2}{3}\%$ of what number? 40%? 60%?
75%? 100%? $16\frac{2}{3}\%$? 20%? 30%?

Find the base in each of the following:

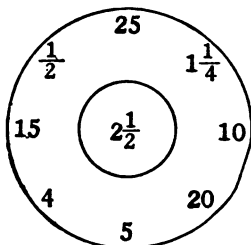
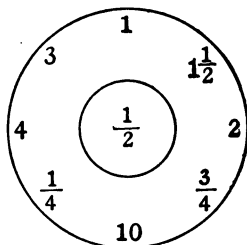
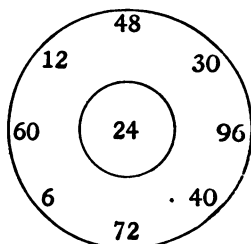
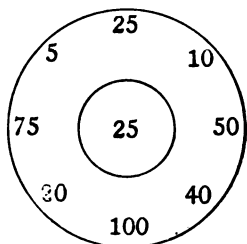
RATE %	PERCENT- AGE	BASE	RATE %	PERCENT- AGE	BASE
3. $66\frac{2}{3}$	24	?	8. 25	150	?
4. 75	87	?	9. 80	72	?
5. $33\frac{1}{3}$	200	?	10. $26\frac{2}{3}$	\$1.60	?
6. $62\frac{1}{2}$	60	?	11. 50	\$2.50	?
7. $87\frac{1}{2}$	56	?	12. 12	\$ 48	?

* Once the analysis is understood, all problems of this type may be solved as examples in division, either of common fractions or of decimals.

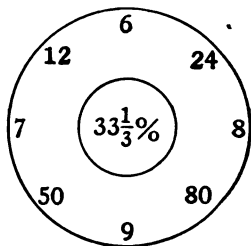
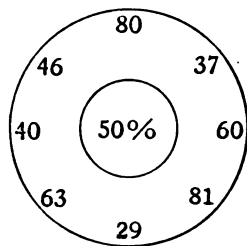
ORAL DRILLS

125.**The Circle**

Find what per cent the number in the center is of each number in the ring.



126 Find the number of which each number in the ring is the per cent given in the center.



EXAMPLES AND PROBLEMS

127. NOTE 1. In all the following examples and problems, there will be given the RATE and the PERCENTAGE to find the BASE.

The general rule is

The base equals the percentage divided by the rate.

NOTE 2. Change the rate to an equivalent common fraction whenever by so doing the work can be shortened.

NOTE 3. Prove or test any example by multiplying the base by the rate; the result should be the percentage.

ORAL PROBLEMS

1. How long is a line if $66\frac{2}{3}\%$ of its length is equal to 10 in.?

2. If $33\frac{1}{3}\%$ of its length is equal to 15 in.?

3. 40 % of the area of a blackboard is 1600 sq. in.
What is the area of the blackboard?

4. 75 % of a boy's weight is 90 lb. How much does he weigh?

5. 80 % of the number of pupils in a class is 32.
How many pupils are in the class?

6. 20 % of the cost of a suit was \$4. Find the cost of the suit.

7. $37\frac{1}{2}\%$ of the lumber needed to make a desk cost \$9. How much would all the required lumber cost?

8. $12\frac{1}{2}\%$ of the distance I had to ride was $2\frac{1}{2}$ mi. What was the entire distance?

9. By working for $62\frac{1}{2}\%$ of a week, a man

earns \$25. How much should he receive for working a whole week?

10. A train goes 20 miles in 40 % of an hour. At that rate how many miles will the train travel in an hour?

WRITTEN EXERCISES

128. 1. $450 = 66\frac{2}{3}\%$ of what number?

2. $8750 = 62\frac{1}{2}\%$ of what number?

3. $87\frac{1}{2}\%$ of what number = 560?

4. $200 = 18\frac{3}{4}\%$ of what number?

Process

$$\begin{array}{r}
 1066\frac{2}{3} \text{ Ans.} \\
 \times 1875. \overline{) 200 \times 0000} \\
 \underline{1875} \\
 12500 \\
 \underline{11250} \\
 12500 \\
 \underline{11250} \\
 1250 \\
 \underline{1875} = \frac{2}{3}
 \end{array}$$

5. $1728 = 12\%$ of what number?

6. Find the number of which $22\% = \$68.20$.

7. Find the number of which $\$187.50 = 45\%$.

8. $32\frac{1}{2}\%$ of what number is 1300?

9. 180 is $1\frac{1}{2}\%$ of what number?

10. $\frac{3}{4}\%$ of what number = 9?

11. $650 = 21\frac{2}{3}\%$ of what number?

Process

$$\begin{array}{r} 3000 \text{ Ans.} \\ .21\frac{2}{3} \overline{)650.00} = \times 65. \overline{)1950 \times 00.} \\ \underline{195} \end{array}$$

12. \$2.40 = $13\frac{1}{3}\%$ of what number?

WRITTEN PROBLEMS

129. 1. 16% of a sum of money equals \$128.
What is the total sum?

2. 28% of a crop of potatoes equals 252 bu.
How many bushels are in the entire crop?

3. If a furnace consumes 8 T. of coal in 22%
of a month, how many tons will it consume in a
month?

4. 82% of a plot of land is worth \$1246.40.
What is the value of the entire plot?

5. The area of 18% of a roadway is 3600
sq. ft. Find the area of the entire roadway.

6. 15% of the expenses of running a factory
amount to \$4560. What are the total expenses of
the factory?

7. 32% of the population of a county is 20,480
people. What is the entire population of the
county?

8. 26% of the cost of a wagon is \$143. Find
the cost of the wagon.

9. 34% of the length of a river is 255 mi. What is its entire length?

10. One candidate for an office receives 6300 votes. This was 35% of the number of votes the other candidate received. How many did both candidates receive?

Second Case

130. 3 (b), To find a number, when that number plus a per cent of it is given.

3 (c), To find a number, when that number minus a per cent of it is given.

1. How much is 20 increased by $\frac{1}{4}$ of 20? How much is 20 increased by $\frac{1}{5}$ of itself? How much is 20 increased by $\frac{4}{5}$ of itself?

2. 24 decreased by $\frac{1}{3}$ of itself equals what number? 24 decreased by $\frac{2}{3}$ of itself equals what number? 24 decreased by $\frac{3}{4}$ of itself equals what number?

3. How much is 20 increased by 25% of itself? 24 decreased by $66\frac{2}{3}\%$ of itself equals what number?

4. If a number is increased by 25% of itself, what per cent represents the sum?

NOTE. The number increased or decreased is the base; it is always represented by 100%.

5. If a number is decreased by $66\frac{2}{3}\%$ of itself, what per cent represents the difference?

6. What number increased by 75% of itself equals 140?

Process

$\frac{3}{4}$ of a number = 140

therefore the number = $\frac{4}{3}$ of $\overset{20}{140} = 80$ *Ans.*

Explanation. Every number equals 100 % of itself.

(100 % equals 1 and may be expressed as $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, $\frac{5}{5}$, $\frac{1}{1}$, or in any other form required. In this example, since $75\% = \frac{3}{4}$, the form required will be $\frac{4}{3}$.)

Since the number was *increased* by $\frac{1}{4}$ of itself,

$\frac{1}{4}$ of the number = 140

$\frac{1}{4}$ of the number = $\frac{1}{4}$ of 140 = 20

$\frac{4}{4}$ of the number = $4 \times 20 = 80$ *Ans.*

Shorter Explanation

$\frac{3}{4}$ of the number = 140, therefore the number equals $\frac{4}{3}$ of 140 = 80 *Ans.*

One Form of Proof

80 increased by 75 % of itself = ?

$\frac{3}{4}$ of $\overset{20}{80} = 60$ $80 + 60 = 140$.

7. What number decreased by 40 % of itself equals 60 ?

Process

$\frac{3}{5}$ of a number = 60

therefore the number equals $\frac{5}{3}$ of $\overset{20}{60} = 100$ *Ans.*

Explanation

Since the number was *decreased* by $\frac{2}{5}$ of itself,

$\frac{2}{5}$ of the number = 60

$\frac{2}{5}$ of the number = $\frac{2}{5}$ of 60 = 20

$\frac{5}{5}$ of the number = $5 \times 20 = 100$ *Ans.*

Shorter Explanation

$\frac{2}{3}$ of the number = 60, therefore the number equals

$$\frac{5}{3} \text{ of } 60 = 100 \text{ Ans.}$$

One Form of Proof

100 decreased by 40 % of itself = ?

$$\frac{2}{3} \text{ of } 100 = 66\frac{2}{3} \quad 100 - 66\frac{2}{3} = 33\frac{1}{3}$$

ORAL EXERCISES

- 131.** 8. 36 is $33\frac{1}{3}$ % more than what number?
 9. 48 is 20 % less than what number?
 10. \$2.25 is $12\frac{1}{2}$ % more than what number?
 11. What number increased by $16\frac{2}{3}$ % of itself equals 42?
 12. What number decreased by 25 % of itself equals \$1.20?
 13. What number increased by 100 % of itself equals 50?
 14. What number increased by 150 % of itself equals 25?
 15. What number decreased by 50 % of itself equals 18?

132. When the given per cent is not easily reducible to a simple common fraction, use the following explanation and solution.

What number increased by 42 % of itself equals 7384?

Process

$7384 = 142\%$ or 1.42 times the number.

$7384 \div 1.42 =$ the number.

$$\begin{array}{r}
 52\ 00 \\
 1.42 \overline{)7384.00} \\
 \underline{710} \\
 284 \\
 \underline{284} \\
 00
 \end{array}
 \quad 5200 \text{ Ans.}$$

One Form of Proof

$5200 + 42\%$ of itself = what number?

$$\begin{array}{r}
 5200 \\
 \times 42 \\
 \hline
 104 \\
 208 \\
 \hline
 2184 \text{ increase}
 \end{array}
 \qquad
 \begin{array}{r}
 5200 \\
 2184 \\
 \hline
 7384
 \end{array}$$

EXAMPLES AND PROBLEMS

NOTE 1. In all the following examples and problems, there will be given the **rate of increase** and the **amount**, or the **rate of decrease** and the **difference**, to find the **base**.

The general rule is:

The base equals the amount divided by 1 plus the rate of increase; or

The base equals the difference divided by 1 minus the rate of decrease.

NOTE 2. Change the rate to an equivalent common fraction whenever by so doing the work can be shortened.

NOTE 3. Prove or test by multiplying the base by the rate; then find the amount or the difference.

133.

WRITTEN EXERCISES

	WHAT NUMBER	INCREASED BY — % OF ITSELF	DECREASED BY — % OF ITSELF	EQUALS
1.	?	15		2760
2.	?	—	45	1100
3.	?	65	—	\$ 39.60
4.	?	$37\frac{1}{2}$	—	2200 tons
5.	?	—	70	\$ 9.45
6.	?	15	—	\$ 69000
7.	?	—	$12\frac{1}{2}$	\$ 1172.50
8.	?	—	35	\$ 32.24
9.	?	$17\frac{1}{2}$	—	\$ 23500
10.	?	50	—	840 bbl.

PROBLEMS

134. 1. A boy lost 8% of his marbles and then had 46 marbles left. How many did he lose?

2. After adding $33\frac{1}{3}\%$ to his stock, a man owns 600 sheep. How many sheep had he at first? How many did he add?

3. After selling 80% of the entire edition of a book, a publisher had 2000 copies still on hand. How many copies did he sell?

4. The population of the United States according to the census of 1910 was about 91,972,100. This represents an increase of 21% over the figures of 1900. What was the population in 1900? What is the increase?

5. The aggregate school attendance of New York City is about 630,000. This shows an increase of 6 % over last year's attendance. What was last year's attendance? What was the increase? What will be next year's attendance at the same rate of increase?

ORAL EXERCISES

135. Find the values for ?

	BASE	RATE % IN- CREASE	RATE % DE- CREASE	INCREASE	DECREASE	AMOUNT	DIFFER- ENCE
1.	60	10	—	?	—	?	—
2.	60	—	10	—	?	—	?
3.	?	—	25	—	?	—	30
4.	?	25	—	?	—	30	—
5.	30	?	—	?	—	40	—
6.	40	—	?	—	?	—	30
7.	?	20	—	12	—	—	—
8.	?	?	$33\frac{1}{3}$	—	16	—	—
9.	?	$66\frac{2}{3}$	—	?	—	60	—
10.	?	—	40	—	—	—	45
11.	20	25	—	?	—	?	—
12.	25	—	20	—	?	—	?
13.	45	$33\frac{1}{3}$	—	—	—	?	—
14.	50	—	10	—	—	—	?
15.	?	—	25	—	10	—	—
16.	30	$66\frac{2}{3}$	—	?	—	?	—
17.	?	$12\frac{1}{2}$	—	10	—	—	—
18.	48	—	75	—	?	—	?
19.	100	100	—	—	—	?	—
20.	?	—	$16\frac{2}{3}$	—	—	—	30

PROFIT AND LOSS

FOR READING

136. Business is based upon operations in which one person sells and another person buys. A profitable or well-paying business is one in which the owner sells the articles for more than he paid for them. A losing business is one in which the owner sells the articles for less than he paid for them. When an article is sold for more than it cost, the owner makes a **profit** or a **gain**; when it is sold for less than it cost, he sustains a **loss**.

Profit and **Loss** is that form of percentage which deals with the gains made or losses sustained in business transactions.

137. The terms used in Profit and Loss are :

	ABBREVIATIONS
Cost or (Buying Price).	C.
Gain per cent or Loss per cent.	G. % or L. %
Gain or Loss (in money).	G. or L.
Selling Price.	S. P.

A man bought a cow for \$60 and sold it at a gain of 25 %. What was his gain? What was the selling price?

$$25\% \text{ (or } \frac{1}{4}\text{)} = \text{G. \%}. \quad \frac{1}{4} \text{ of } \$60 = \$15. \quad \text{G. in money.} \quad \begin{array}{r} \$60 \text{ C} \\ 15 \text{ G.} \\ \hline \text{S. P. } \$75 \end{array}$$

If he had sold the cow at a *loss* of 25 %, what would have been the loss? The selling price?

$$25 \% \text{ (or } \frac{1}{4}) = \text{L. } \%. \quad \frac{1}{4} \text{ of } \$60 = \$15 \quad \text{L. in money.} \quad \$60 \text{ C.}$$

$$\begin{array}{r} 15 \text{ L.} \\ \hline \text{S. P } \$45 \end{array}$$

Remember that:

When goods are sold at a *loss*, the selling price is less than the cost.

When goods are sold at a *gain*, the selling price is more than the cost.

EQUIVALENT TERMS

PERCENTAGE	PROFIT AND LOSS
Base	Cost
Rate per cent	G. or L. per cent
Percentage	G. or L. in money
Amount	Selling Price
or	
Difference	

TO FIND THE GAIN OR LOSS, THE COST AND PER CENT OF GAIN OR LOSS BEING GIVEN

ORAL EXERCISES

138. Find, at sight, the number indicated by ? in each of the following examples. (Prove the odd-numbered examples in each exercise.)

	COST	GAIN PER CENT	LOSS PER CENT	GAIN	LOSS	SELLING PRICE
1.	\$60	50	—	?	—	?
2.	\$72	$33\frac{1}{3}$	—	?	—	?
3.	\$80	—	10	—	?	?
4.	\$2.40	—	$16\frac{2}{3}$	—	?	?
5.	\$6.50	100	—	?	—	?
6.	\$500	—	20	—	?	?
7.	\$480	25	—	?	—	?
8.	\$1000	—	30	—	—	?
9.	\$1.25	—	20	—	?	?
10.	\$7.50	—	$33\frac{1}{3}$	—	—	?
11.	\$9000	40	—	?	—	?
12.	\$5000	100	—	?	—	?

139.

WRITTEN EXERCISES

	COST	GAIN PER CENT	LOSS PER CENT	GAIN	LOSS	SELLING PRICE
1.	\$25,000	$18\frac{3}{4}$	—	—	—	?
2.	\$ 4,800	$22\frac{1}{2}$	—	?	—	?
3.	\$ 7,964	—	25	—	?	?
4.	\$ 2,500	—	3	—	?	?
5.	\$ 8,750	$2\frac{1}{2}$	—	?	—	?
6.	\$10,000	4	—	—	—	?
7.	\$ 8,400	—	50	—	?	?
8.	\$18,750	—	11	—	?	?
9.	\$16,000	—	$31\frac{1}{3}$	—	—	?
10.	\$ 1,728	100	—	—	—	?

WRITTEN PROBLEMS

140. 1. An agent bought 48 bbl. of apples at \$1.10 per barrel and sold them at a gain of 30 %. Find his total gain.

2. 10 harvesting machines cost \$285 each. They are sold at a gain of 14 %. Find the gain. Find the selling price.

3. The total production of a farmer's wheat field is 2250 bu. At first he intended to sell the wheat for 90 ct. a bushel, but later he sold it for 12 % more. How much did he receive for the entire crop?

4. 800 bbl. of apples were bought for \$1500. The purchaser sold them at a gain of 16 %. Find his gain on each barrel.

5. 70 head of cattle were bought at an average price of \$95 per head. They were sold at a loss of 15 %. Find the total loss.

6. 250 wooden toys are made at a cost of 60 ct. each. They are sold at a gain of 15 %. Find the total amount received for the toys.

7. A storekeeper buys 360 baseball gloves for 80 ct. each. He sells $\frac{1}{4}$ of them at a gain of 10 % ; $\frac{1}{2}$ of them at a gain of 20 % ; and the remainder at a loss of 10 %. Find the total gain or loss on all the gloves.

8. A gas company makes gas for $62\frac{1}{2}$ ct. per

1000 cu. ft. It sells the gas at a gain of 28 %. Find the profit on 75,000 cu. ft.

9. A baker made 8000 loaves of bread at 4 ct. a loaf. He could not sell $12\frac{1}{2}\%$ of the loaves and gave them away. The remainder were sold at a gain of 20 %. What was the baker's total gain or loss in money?

10. A farmer sent 225 baskets of peaches to the city. The peaches cost him 80 ct. a basket. 20 % of the peaches became damaged and were thrown away; the remainder were sold at a gain of $37\frac{1}{2}\%$. How much did the farmer gain or lose?

11. 144 doz. pairs of stockings were bought at 25 ct. a pair. They were sold at a gain of 16 %. Find the total gain.

12. 150 ladies' waists were sold to a storekeeper for \$4.40 each. The storekeeper sold $\frac{1}{3}$ of the waists at a gain of 60 % ; $\frac{1}{2}$ of them at a gain of 20 % ; and the remainder at \$3.50 each. How much did the storekeeper gain or lose?

13. Mr. Williams bought a house for \$15,000. He kept it for one year, spending \$175 for taxes and \$420 for repairs. Then he sold the house at a net gain of 10 %. What was the selling price?

14. A firm manufactures 20 automobiles at an average cost of \$3400 each. The automobiles are sold at an average gain of $22\frac{1}{2}\%$. Find the gain.

15. A dry goods merchant bought 25,000 yd. of silk at \$1.75 a yd., and sold it at a gain of 21 %. Find his gain.

16. 95 auto trucks were manufactured at a cost of \$750 each; they were sold at a gain of 16 %.

(a) Find the gain on each truck.

(b) Find the total gain.

**TO FIND THE RATE OF GAIN OR LOSS, THE
COST AND SELLING PRICE BEING GIVEN**

41. The per cent of gain or loss is always reckoned on the cost or on the sum invested in the transaction.

I bought for \$8 and sold for \$12. What is the gain per cent?

Explanation. The buying price or cost is \$8; the selling price is \$12; the gain is the difference or \$4. A gain of 4 on a cost of \$8 = $\frac{4}{8}$ or $\frac{1}{2}$ = 50 % G. *Ans.*

A desk that cost \$20 was sold for \$18. Find the loss per cent.

Explanation. The loss is \$2 on a cost of \$20. Therefore, the loss % is $\frac{2}{20} = \frac{1}{10}$ = 10 % loss. *Ans.*

ORAL EXERCISES

42. In the following examples, find as many of the missing terms as you can.

	COST	SELLING PRICE	GAIN IN MONEY	LOSS IN MONEY	GAIN %	LOSS %
1.	\$6	\$8	\$—	\$—	—	—
2.	8	6	—	—	—	—
3.	10	9	—	—	—	—
4.	9	10	—	—	—	—
5.	5	10	—	—	—	—
6.	10	5	—	—	—	—
7.	9	27	—	—	—	—
8.	27	9	—	—	—	—
9.	200	250	—	—	—	—
10.	250	200	—	—	—	—
11.	24	30	—	—	—	—
12.	30	24	—	—	—	—
13.	60	120	—	—	—	—
14.	120	60	—	—	—	—
15.	48	—	—	12	—	—
16.	—	48	—	12	—	—
17.	—	75	25	—	—	—
18.	—	75	—	25	—	—
19.	—	24	—	—	—	50
20.	—	48	—	—	—	33½
21.	30	—	—	—	25	—
22.	60	50	—	—	—	—
23.	—	60	10	—	—	—
24.	60	—	—	10	—	—
25.	—	60	—	—	25	—
26.	—	60	—	—	—	25
27.	60	—	—	—	25	—
28.	60	—	—	—	—	25
29.	60	—	20	—	—	—
30.	—	—	60	—	10	—

MARKING GOODS

FOR READING

143. Merchants often mark goods in *cipher*. This means that they use signs or letters to represent the cost and the selling price of goods. If letters are chosen, the *key* is a word of ten different letters to be used instead of the ten numerals. Sometimes two words are used instead of one. In order to make the reading of these prices more difficult sometimes a different letter is used when a figure is repeated.

Take as a key word

W a s h i n g t o x*
1 2 3 4 5 6 7 8 9 0

An article costs \$1.25. Its cost ~~mark~~ would be *wai*. If it cost 44 cents, it might be marked *hh*; but if a repeater is used, 44 cents would be written *hz* or *hl*, or any other letter not in the word *Washingtox* which has been selected as a repeater.

If an article bears two marks, the first is the cost price, the second the selling price.

Using *Washingtox* as a key, and "f" as a repeater, an article costing \$2.55 and marked to sell at \$3.25 would be marked $\frac{wif}{sai}$.

* Use x for 0 because n is used for 6.

144. Still using *Washington* as a key, and *f* as a repeater, give the cost and selling price of articles marked as follows:

- | | | |
|-----------------------|------------------------|----------------------|
| 1. $\frac{na}{gi}$ | 2. $\frac{wox}{afi}$ | 3. $\frac{wnh}{wti}$ |
| 4. $\frac{tf}{wxf}$ | 5. $\frac{sa}{hx}$ | 6. $\frac{wix}{axf}$ |
| 7. $\frac{sgi}{hix}$ | 8. $\frac{as}{ao}$ | 9. $\frac{hi}{nx}$ |
| 10. $\frac{iai}{ntx}$ | 11. $\frac{oxf}{waxf}$ | 12. $\frac{tg}{oi}$ |

145. Take *vanderbilt* as a keyword, using *o* as a repeater. Write out in cipher the cost of the following items and the selling price at 20 % gain.

	COST	COST	SELLING PRICE
1.	\$ 1.75	_____	_____
2.	\$ 2.50	_____	_____
3.	\$.45	_____	_____
4.	\$ 1.25	_____	_____
5.	\$ 7.50	_____	_____
6.	\$.35	_____	_____
7.	\$ 1.10	_____	_____
8.	\$ 7.00	_____	_____
9.	\$ 8.40	_____	_____
10.	\$ 12.50	_____	_____

11. Select a keyword of your own and construct a table of cost and selling price at $33\frac{1}{3}$ % of profit.

COMMISSION AND BROKERAGE

FOR READING

146. Many people employ agents to buy or sell merchandise, property, or other marketable goods, or to transact other business for them. Such agents are called **commission merchants** or **brokers**.

NOTE 1. These terms are often used interchangeably; there is, however, a difference in their meaning. A commission merchant actually receives the goods from a merchant or principal, or forwards the goods to another merchant; while the broker makes the arrangements for purchase only, allowing the owner to ship or send direct to the purchaser.

'Commission or brokerage is the amount of money paid to an agent for buying or selling goods.

NOTE 2. In buying, commission is reckoned on the cost, or **NET COST** as it is sometimes called. The net cost plus the commission and other charges, such as freight, cartage, insurance, etc., is called the **GROSS COST**. In selling, commission is reckoned on the amount of the sale. The rate of commission or brokerage is expressed in per cent. The sum of money left after deducting the commission and other expenses is called the **NET PROCEEDS**.

EQUIVALENT TERMS

PERCENTAGE	PROFIT AND LOSS	COMMISSION OR BROKERAGE
Base	Cost	Cost or Selling Price
Rate	Gain or Loss %	Rate of Commission
Percentage	Gain or Loss (\$)	Commission or Brokerage
Amount	Selling Price	Total Cost
or		or
Difference		Net Proceeds

The processes and explanations of problems are the same as in Percentage.

TO FIND THE COMMISSION, THE COST OR SELLING PRICE, AND THE RATE OF COMMISSION BEING GIVEN

RULE. The commission equals the base multiplied by the rate.

147.

ORAL EXERCISES

		COST OR SELLING PRICE	RATE OF COMMISSION	TOTAL COST	NET PROCEEDS
1.	Bought	\$ 5,000	2 %	?	
2.	Sold	\$ 7,500	$1\frac{1}{2}$ %	—	?
3.	Bought	\$ 6,400	$1\frac{1}{4}$ %	?	
4.	Bought	\$ 10,000	$\frac{1}{2}$ %	?	
5.	Sold	\$ 9,600	5 %	—	?
6.	Sold	\$ 7,200	$\frac{1}{3}$ %	—	?

WRITTEN EXERCISES

148. 1. A real estate broker sells property for \$25,000. His commission is $1\frac{1}{2}$ %. How much does he receive?

2. A book agent sells 100 sets of books at \$165 per set. Find his commission at 16 %.

3. An agent sells 20 sewing machines for \$45 each. His commission is $16\frac{2}{3}$ %. Find his commission.

A commission house makes the following sales.
Find the commission in each case.

	ARTICLES	PRICE	RATE OF COMM.	COMM.
4.	250 bbl. apples	\$ 1.45 per bbl.	10 %	?
5.	200 bbl. potatoes	1.15 per bbl.	8 %	?
6.	200 bbl. cabbages	4.20 per bbl.	$7\frac{1}{2}$ %	?
7.	500 bbl. flour	5.50 per bbl.	15 %	?
8.	120 tubs of butter averaging $52\frac{1}{2}$ lb.	22 ct. per lb.	12 %	?
9.	10 cases eggs of 30 doz. each	24 ct. per doz.	$6\frac{2}{3}$ %	?
10.	100 boxes crackers	\$ 2.10 per box	$14\frac{3}{4}$ %	?
11.	80 crates tomatoes	75 ct. per crate	$12\frac{1}{2}$ %	?
12.	200 baskets peaches	80 ct. per basket	20 %	?

A real estate broker made the following sales.
Find the brokerage in each case.

	PROPERTY	PRICE	RATE OF BROKER- AGE	BROKER- AGE
13.	House	\$ 12,500	$1\frac{1}{2}$ %	?
14.	Farm, complete	\$ 16,000	5 %	?
15.	Factory	\$ 120,000	$2\frac{1}{2}$ %	?
16.	Office-building	\$ 225,000	$1\frac{1}{4}$ %	?
17.	10 lots	\$ 450 each	3 %	?
18.	House	\$ 27,500	$1\frac{3}{4}$ %	?
19.	Furnished House	\$ 32,000	$2\frac{1}{4}$ %	?
20.	5 Houses	\$ 7500 each	2 %	?

TRADE DISCOUNT

FOR READING

149. Many manufacturers issue catalogs and price lists in which their goods are marked at the very highest price at which such goods can be sold to retail dealers. Jobbers and retail merchants must buy for a lower price than the list price, or they cannot afford to deal in such goods. For instance, if sewing machines are listed for \$ 40 each, a dealer could not profitably sell the machines at that price, unless he could buy them for less than \$ 40. The manufacturer is willing to sell machines to the dealer subject to several discounts. The condition of the market and the quantity bought usually regulate the amount of discount given.

Merchants who give large discounts seldom quote one *rate*, but offer a *series* of discounts. The manufacturer might sell the \$ 40 machine, subject to discounts of 40, 10, and 5. This means, that he will deduct 40 % from the list price, 10 % from the remainder, and 5 % from the second remainder. The third remainder is the amount the dealer pays for the machine. He can then sell the machine for \$ 40 and make a profit.

The price at which an article is sold after all discounts have been subtracted is called the **net price** or **net cost**.

Explanation. \$40 is the list price. 40 % of \$40 = \$16. This leaves \$24 as the price after one discount has been deducted. The second discount is 10 %. 10 % of \$24 is \$2.40, which deducted from \$24 leaves \$21.60, as the price after the second discount has been deducted. The last discount is 5 %. 5 % of \$21.60 is \$1.08, which deducted from \$21.60 leaves \$20.52 as the NET PRICE or NET COST.

Discounts are deducted in succession, but it is immaterial in which order the discounts are deducted, as it will not affect the result.

Process and Explanation. (Usual Method)

\$ 40	List price.
16	40 % discount.
<u>\$ 24</u>	1st remainder.
2.40	10 % discount.
<u>\$ 21.60</u>	2d remainder.
1.08	5 % discount.
<u>\$ 20.52</u>	net cost or net price. <i>Ans.</i>

Second Method. Instead of multiplying the list price by the per cent and deducting the result, it is shorter to deduct the per cent from 100 %, which gives the result at once.

Process and Explanation. (Second Method)

\$.40	List price.
.60	40 % discount leaves 60 % to be paid.
<u>\$ 24.00</u>	1st remainder.
.90	10 % discount, leaves 90 % to be paid.
<u>\$ 21.60</u>	2d remainder.
.95	5 % discount, leaves 95 % to be paid.
<u>1080</u>	
1944	
<u>\$ 20.520</u>	net cost or net price. <i>Ans.</i>

There is a shorter way.

Discounting 40 %, 10 %, and 5 % means selling at 60 %, 90 %, and 95 % of the list price; expressed decimally .6, .9, .95.

Multiply these in succession, $.6 \times .9 \times .95 = .5130$.

List price \$40

Net price per dollar .513

\$20.52 net price.

WRITTEN EXERCISES

	ARTICLE	LIST PRICE	% 1ST DISCOUNT	% 2D DISCOUNT	% 3D DISCOUNT	REQUIRED
1.	1 Piano	\$600	33½	10	2	Total Discount
2.	1 Piano	\$480	25	10	6	Net Price
3.	1 Automobile	\$3,000	30	20	10	3d Discount
4.	1 Automobile	\$2,750	20	15		Net Price
5.	3 Books	@ \$1.50	20	10	2	Total Discount
6.	6 Books	@ \$4.50	20	10	2	Net Price
7.	2 Sewing Machines	@ \$60	30	10		2d Discount
8.	1 Sewing					
9.	Machine	\$75	33½	10	6	Total Discount
	2 Rugs	@ \$125	20	10		Net Price
10.	Brushes per dozen	\$4.50	16½	10	10	Net price per brush

SPECIAL DISCOUNTS

150.

FOR READING

Goods are often sold without trade discounts, but with an allowance for paying cash. A deduction of this kind is called a **cash discount**. Some dealers allow a **CASH DISCOUNT** in addition to trade discounts.

Merchants often buy goods to be paid for in 30, 60, 90, or 120 days. Manufacturers will offer a special discount for earlier payment. Some houses offer 10 % for cash, some 7 %, some 5 %, most 2 %.

Dressmakers and shopping agents often receive a special discount on purchases made for other people. This custom prevails in order that those who are finally to receive the merchandise pay no more for the goods, but the agent receives the amount of the discount to cover the trouble and expense of shopping.

PROBLEMS

151. 1. Mr. Jones owes \$350. He is allowed a discount of 6 % for prompt payment. How much does he save by paying the bill at once?

2. A plumber bought \$250 worth of goods. He was allowed a discount of 3 % for cash. How much did the goods cost him?

3. A merchant buys \$800 worth of lace. He is allowed a discount of $4\frac{1}{2}$ % for cash or 2 % if paid within 30 days. He waits for 30 days before paying. How much did he lose by not paying cash?

4. A workingman buys a home for \$3000, with a discount of 4 % if he pays within a year. He can get a discount of 10 % for cash. How much will he gain if he pays the whole amount at once?

5. A painter buys 60 rolls of wall paper. The

list price is \$3.50 a roll. From this price, the painter gets a discount of 8% and an additional 2% for cash. How much did the wall paper cost him?

An automobile catalog quotes discounts of $33\frac{1}{3}\%$ and 10%.

Find the net price of each of the following machines:

	LIST PRICE	NET PRICE	($33\frac{1}{3}$ and 10 off.)
6.	\$4500	?	
7.	2250	?	
8.	1400	?	
9.	6500	?	
10.	7250	?	

A wholesale stationery house allows discounts of 20%, 10%, and 5%.

Find the net cash price of each of the following:

11. 1 gross lead pencils listed at 72 ct. a dozen.
12. 2 gross wooden penholders listed at 20 ct. a dozen.
13. 6 doz. ink wells listed at 15 ct. each.
14. 10 doz. blank books, style *A*, listed at 35 ct. a dozen.
15. 150 blank books, style *B*, listed at 60 ct. a dozen.
16. Make out a separate bill for each Example, from 11 to 15 inclusive.
17. Make out one bill for all the goods.

REVIEW OF TYPE CASES IN COMMON FRACTIONS, DECIMALS, AND PERCENTAGE

Type I

152. 1. (a) Find $\frac{3}{4}$ of 120.

(b) How much is .75 of 120?

(c) Sold 75% of 120 eggs. Find the number of eggs sold.

Type II

2. (a) Had 35 ct.; spent 21 ct. What part of my money did I spend?

(b) 21 is how many .00's of 35?

(c.) Out of 35 books, 21 were old. What per cent of the books were old?

Type III (1st Case)

3. (a) A boy was absent 4 days. This was $\frac{1}{20}$ of the term. How many days were in the term?

(b) $4 = .05$ of what number?

(c) 5% of a class is absent. 4 boys are absent. What is the register of the class?

Type III (2d Case)

4. (a) 25 is $\frac{1}{4}$ more than what number?

(b) A farmer has 25 sheep; this is .25 more than he had at first. How many sheep had he at first?

(c) I sold a watch for \$ 25. This was 25 % more than it cost. Find the cost.

Type III (3d Case)

5. (a) John is 18 yr. old ; this is $\frac{1}{5}$ more than his sister's age. What is his sister's age ?

(b) $18 = .20$ more than what number ?

(c) After increasing my stock of pencils 20 % , I had 18 pencils. How many pencils had I at first ?

MISCELLANEOUS PROBLEMS

153. Use pencils only when the numbers are too large to be carried in the mind.

1. 24 out of the 60 girls in a class were marked excellent. What per cent of the class received an excellent mark ?

2. A boy has \$25 in the bank. 16 % of it he earned. The rest his father gave him. How much money did his father give him ?

3. In a class of 50 boys, 6 % were absent. How many boys were absent ?

4. In a class of 50 boys, 74 % were absent. How many boys were present ?

5. A coat costing \$24.50 was sold at a loss of \$3.50. What was the loss per cent ?

6. A class had a library of 864 books. $37\frac{1}{2}$ % of the books are story books. How many story books are there in the library ?

7. In a school of 1250 pupils, 1225 were present. What per cent of the school was present?

8. In a school of 1200 pupils, 3 % were late. How many pupils came late?

9. Tea that cost 40 ct. a pound is sold for 55 ct. a pound. Find the per cent gained?

10. What is the difference on a bill of \$760 between a direct discount of 25 % and discounts of 15 % and 10 %?

11. A ton of coal costing \$3.50 is sold for \$7. Find the gain per cent.

12. A man owning 80 % of a mill sold $62\frac{1}{2}$ % of his share for \$4000. At that rate what was the value of the mill?

13. What is the cost of 12 yd. of cloth, if 28 % is gained by selling it at \$2.56 a yard?

14. I bought a bill of goods for \$218 at 10 % discount and 8 % off for cash. What was the net cost?

15. John is 36 years old. His wife's age is 75 % of his age. How much older is he than his wife?

16. The list price of a shipment of goods is \$750. The buyer received discounts of 20 and $12\frac{1}{2}$ %. What did the goods cost him?

17. A man spends 80 % of his salary and has \$350 left. What is his salary?

18. A piano which cost \$275 was sold for \$225. What was the loss per cent?
19. A manufacturer sold a carriage for \$432, which was 20 % above cost. What was the cost?
20. 75 % of \$1000 = ? per cent of \$4000?
21. \$300 is what per cent of \$300?
22. I sold 2 horses for \$170 each. On one I gained 20 %, on the other I lost 20 %. What was the gain or loss in money on the entire transaction?
23. A house worth \$7500 was insured for $\frac{2}{3}$ of its value at $\frac{3}{5}$ %. What was the premium, or annual payment?
24. The tax rate is $1\frac{1}{2}$ %. How much must be paid in taxes on property worth \$4000?
25. An agent sold 400 yd. of cloth at \$3 a yard. How much was his commission at 5 %?
26. A collector succeeded in collecting 75 % of a debt of \$4000. What is his commission at $8\frac{1}{3}$ %?
27. A man paid \$4500 for a house. He spent \$150 repairing it and then sold it for 18 % above cost. What did he receive for it?
28. A merchant lost $37\frac{1}{2}$ % when he sold goods for \$82.20. What did the goods cost?
29. At $33\frac{1}{3}$ % what is the duty on an invoice of muslins amounting to \$3690?
30. The duty on silk is 40 %. What amount of duty must be paid on \$19,500 worth of silk?

31. A newsdealer ordered 200 *Worlds*, 175 *Times*, 250 *Journals*, 125 *Heralds*, 75 *Presses*. What per cent of his papers were *Worlds*? *Times*? *Journals*?

32. There were 90 school days in a term. A boy was present 80% of the term. How many days did he lose?

33. A department store took in \$8910 on Saturday. This was 33% of the week's receipts. How much was taken in during the entire week?

34. A house is worth \$7880. For what amount must it be insured to cover 90% of its value?

35. A boy gave his brother 20% of his marbles, and lost 5% of the remainder. He then had 152 marbles. How many had he at first?

36. A grocer bought 15 tubs of butter, each containing 56 lb., at 28 ct. per pound, and sold them at 35 ct. per pound. What was his gain %? (Use short method.)

37. A merchant lost \$382.20 by selling goods at a loss of 9%. What did the goods cost?

38. A salesman received \$180 for selling goods at $21\frac{1}{2}\%$. What amount of goods did he sell?

39. Cloth costs $87\frac{1}{2}$ ct. a yard. For how much must it be sold to gain 30%?

40. School desks listed at \$16.50 are sold at 20 and 10 off. What will they cost?

41. A man earns \$18 a week and spends $\frac{1}{4}$ of it

for board and $\frac{1}{8}$ of it for other expenses. What per cent of his wages does he spend? (Use short method.)

42. What is the loss per cent on a dozen eggs when they are bought at 3 for 14 ct. instead of at 50 ct. per dozen?

43. When butter is 42 ct. per lb., what per cent does the dealer gain by selling it at 2 oz. for 7 ct.?

44. What will it cost to insure a house valued at \$4875, for three years, at $\frac{3}{4}\%$ per annum?

45. A store valued at \$5000 was destroyed by fire. The owner received \$3500 insurance. What was his loss per cent?

46. For how much must apples costing \$2.40 a barrel be sold to bring a gain of $37\frac{1}{2}\%$?

47. A boy buys apples at the rate of 5 for 5 ct., and sells them at the rate of 3 for 5 ct. What is his per cent of gain?

48. A and B form a partnership, A investing \$1800, B \$2400. What per cent of the profits will each receive?

49. $12\frac{1}{2}\%$ of the 312 pages in a magazine are filled with advertisements. How many pages are left for reading matter?

50. A suit of clothes costs \$12.50. At what price must it be sold so that the merchant may gain 20%?

51. An overcoat costs \$16. At what price must it be marked in order that the merchant may give a discount of 20 % and still gain 25 % of the cost price?

52. There are 192 boys or 12 % of the entire school on register in the 6th grade. How many boys in the entire school?

53. The retail price (list price) of a dozen teaspoons is \$4.80. The wholesale price is 40 % off. What will be the cost of 12 doz. teaspoons at the wholesale price, allowing 2 % discount for cash?

54. Coal sells at \$6.50 a ton or \$.40 a cwt. What per cent is lost by buying a ton by the cwt.?

55. A coal dealer pays \$6.25 for a ton of coal. He sells it at 40 ct. a cwt. What per cent does he gain?

56. I buy neckties at \$8.40 a dozen. I sell them at a gain of $14\frac{2}{7}$ %. What do I receive for each tie?

57. A stationer buys a gross of colored pencils for \$2.88. He sells them at the rate of 2 for 5 ct. What is his gain per cent?

58. A junk dealer pays 1 ct. a pound for old iron. He sells it by the ton and makes 35 % profit. How much does he get for a ton?

59. The rate of taxation being $1\frac{1}{2}$ %, how much

must a company pay whose property is assessed at \$75,000?

60. I owned land worth \$15,000. It was assessed for $83\frac{1}{3}\%$ of its value. What was the tax at $2\frac{1}{5}\%$?

61. A man's salary is \$75 a month. If he spends 65% of it each month, how much will he have at the end of 6 mo.?

62. There are 300 working days in the year. Brown works 6 da. each week for 44 wk. What per cent of the working time is he idle?

63. \$33.75 is 9% of what amount?

64. A collector succeeded in collecting $37\frac{1}{2}\%$ of a debt of \$14,000. What is his commission at 20%?

65. I sold goods for \$375; they cost \$525. What per cent did I lose?

66. Out of 300 problems given during a term, a boy had 25 wrong. What per cent did he have right?

67. A boy was absent $12\frac{1}{2}\%$ of a school term and was present 91 days. How many days in the term?

68. A dealer bought 28 yd. of cloth at \$1.25 per yard. He sold it at a loss of 15%. What was the selling price per yard?

69. What is the difference between $\frac{1}{4}\%$ of 80 and 25% of 80?

70. A man gained $9\frac{1}{2}\%$ by selling a house for \$228 more than it cost him. What was the cost of the house?

71. What is the net cost of goods marked \$1200 with discounts of 25, 20, and 10?

72. In a school of 480 pupils $43\frac{3}{4}\%$ were boys. How many were girls?

73. 7 in. are what per cent of $3\frac{1}{2}$ yd.?

74. 6 sq. in. are what per cent of a 6-in. square?

75. I bought coal at \$4.75 per long ton, and sold it at \$5.50 per ordinary ton. What is the gain in money on 100 long tons? What is the gain per cent on the transaction?

76. 6 % of \$950 is how much greater than 8 % of \$550?

77. John is 40 % older than Louis. If John is 35 yr. old, how old is Louis?

78. 10 % of a bank deposit was in coin, 18 % was in bills, the balance was in checks. If the bills amounted to \$72, what was the total amount of the deposit? What amount in coin? What amount in checks?

79. 120 sq. rd. are what per cent of an acre?

80. I sold a farm for \$18,750 and lost $6\frac{1}{4}\%$. What was the loss in money?

Percentages of Class Attendance

154. A school has 40 classes.

The first column shows the class.

The second column shows the average monthly register of each class.

The third column shows the average monthly attendance.

Find the per cent of absenteeism in each class.

CLASS	AV. REG.	AV. ATT.	CLASS	AV. REG.	AV. ATT.
1	32	30	21	46	44
2	36	33	22	47	45
3	37	35	23	48	45
4	38	35	24	49	42
5	35	34	25	50	46
6	39	36	26	39	34
7	40	38	27	41	41
8	38	36	28	42	38
9	41	38	29	40	36
10	42	40	30	42	36
11	40	39	31	44	42
12	43	40	32	46	45
13	44	40	33	48	42
14	42	41	34	50	42
15	39	35	35	48	46
16	44	41	36	44	43
17	45	40	37	40	37
18	46	42	38	50	46
19	45	42	39	48	46
20	50	47	40	47	43

REVIEW PROBLEMS IN PERCENTAGE

SECOND SET

155. These problems are intended for advanced pupils. Use paper and pencil only when necessary.

A girl's expenses at college amount to \$500 for the first year. It is expected that there will be an increase of 10 % each year on the expense account of the year immediately preceding.

1. What amount will be needed to pay the expenses of the second year?

2. What amount will be needed for the third year?

3. What amount will be needed for the fourth year?

4. Find the total expenses for the four years.

5. In 1900 the population of the state of Washington was 518,100. In the 10 yr. following, the population increased 120.4 %. What was the population in 1910?

6. In 1900, a manufacturer sold \$200,000 worth of goods. In 1905, he sold 14 % more than in 1900. In 1910, he sold 20 % more than in 1905.

(a) Find the amount of his sales in 1910.

(b) How much more did he sell in 1910 than in 1900?

7. The number of bushels of apples produced in one county in 1905 was 12 % greater than the

number of bushels produced in 1900; the number produced in 1910 was 15 % greater than the number of bushels produced in 1905.

If the crop produced in 1910 equaled 450,800 bushels, how many bushels were produced in 1905? in 1900?

8. A dealer offers a reduction of $12\frac{1}{2}\%$ on the price of a piano listed at \$475. How much money is needed to make the purchase?

9. The cash price of a certain suit of dining-room furniture is \$250. The instalment price is 10 % more. I decide to buy on the instalment plan and agree to pay 10 % at once. How much is my first payment?

10. A city house worth \$15,000 is appraised by the tax bureau at 75 % of its value. What must the owner pay, if the tax rate is 7.8 %?

11. How many yards in $37\frac{1}{2}\%$ of a mile?

12. 28 % of the ore taken from a mine is pure iron. How many pounds of iron can be obtained from a ton of such ore?

13. After reducing the regular price 25 %, a clothier sold an overcoat for \$18. What was the regular price?

14. 75 % of 360 is $66\frac{2}{3}\%$ of what number?

15. The duty on undecorated glassware is 55 %. How much duty must an importer pay on an invoice of \$645?

16. A boy owned a 75 % share of a motor boat and sold $\frac{1}{2}$ of his share for \$150; at this rate, what was the value of the motor boat?

17. A merchant sold 1750 yd. of silk from an invoice of 2800 yd. What per cent of the lot remained?

18. A health inspector condemned 45 % of a lot of eggs. The remainder, amounting to 640 crates, were passed as fresh. How many crates were in the entire lot?

19. A house which is listed at \$8400 can be bought for \$7800. What per cent of the listed price is the reduction?

10. A commission merchant sold 15 % of a consignment of peaches and had 595 baskets left. How many baskets in the consignment?

21. 210 sheep in a flock of 800 died in a snow-storm. What per cent of the flock remained?

22. A capitalist invested 60 % of his capital in U. S. bonds, and 75 % of the remainder in real estate. The rest of his capital, amounting to \$18,000, he loaned on notes. What was his entire capital?

23. A and B are joint owners of a hotel, A's share being $33\frac{1}{3}$ % more than B's. If the hotel is sold for \$80,000, how much should A receive?

24. A speculator bought 400,000 bu. of wheat at

\$1.20 a bushel. He sold 150,000 bu. at a profit of 25%, and the remainder at a loss of 20%. What was his gain or loss on the entire transaction?

25. A college student has an allowance of \$600 a year. His tuition costs \$150; room and board \$240; clothing \$75. The remainder he uses for miscellaneous expenses. Find what per cent of his allowance he spent for each of the four items mentioned.

26. The records of the champions in the National Baseball League for the years 1906 to 1910 are as follows:

YEAR	CHAMPION	GAMES	
		Won	Lost
1906	Chicago	116	36
1907	Chicago	107	45
1908	Chicago	95	55
1909	Pittsburg	110	42
1910	Chicago	104	50

What per cent of the games played in each year was won?

27. A mechanic saves \$5.50 a week, which is $16\frac{2}{3}\%$ of his wages. What are his weekly earnings?

28. I owe my brother 20% more than the sum due my grocer. My debt to both is \$55. What is my debt to each?

29. A merchant can buy two motor trucks for \$3750. One truck is worth 50 % more than the other. What is the price of each ?

30. A truck loaded with merchandise weighs 37,000 lb. which is 85 % more than the truck unloaded. What is the weight of the merchandise ?

31. The register of a certain school increased in one year from 1250 to 1625. The average attendance increased from 1220 to 1515. What was the rate per cent of increase in register and in average attendance ?

32. 300 girls took an examination in arithmetic; 45 failed. What per cent of the whole number passed ?

33. Mr. A, a mason, agreed to build for \$350 a brick wall, 75 ft. long, and of a given height and thickness. The materials were to be supplied by his employer. The mason, becoming ill after he had finished 42 ft. of the wall, assigned his contract to Mr. B, another mason, on the same terms. How much should the employer pay to each mason, upon the completion of the job ?

34. A man pays \$1080 for a horse and wagon. The wagon is worth 40 % more than the horse. What is the value of each ?

35. A lawyer's income for the last year was \$4200. This was $14\frac{2}{3}$ % less than the preceding

year. What was his income for the preceding year?

36. An attorney collected 85% of a debt of \$1250. He charged $12\frac{1}{2}\%$ commission and \$8.50 for disbursements. What amount should he remit to his principal?

37. A produce merchant sold a consignment of flour for \$875 on credit. He charged $2\frac{1}{2}\%$ commission, and $2\frac{1}{2}\%$ for guaranteeing payment by the purchaser. What amount should he remit to his principal?

38. The net proceeds of a sale of cotton were \$8,330. What were the gross proceeds, the broker's commission being 2%?

39. A factor in San Francisco purchased \$40,000 worth of raw silk for a manufacturer in Paterson, N.J. What was the factor's bill for silk and commission, the latter being at the rate of $1\frac{7}{8}\%$?

40. A dealer bought a crop of 600 bu. of potatoes for \$600. He paid \$48.60 for cartage and freight. He sold the potatoes at \$1.10 a bushel. What was the rate per cent of profit on the transaction?

41. A merchant bought linen napkins for \$2.50 per dozen, and sold them for \$3.60 per dozen. What was his gain per cent?

42. How must I sell books which cost \$1.12 each, in order to gain $37\frac{1}{2}\%$?

43. A furniture dealer buys chairs at \$ 4.20 each. He sells a dozen for \$ 75. What is his gain per cent?

44. By selling velvet at \$ 2.25 a yard, a merchant loses 10 %. What did he pay per yard for the velvet?

45. A merchant bought at auction 150 alarm clocks for \$ 95. He sold 100 of them for 75 ct. each, and the remainder for 50 ct. each. What was the gain or loss per cent?

46. If a milliner sells a hat for 50 ct. less than cost, she loses $12\frac{1}{2}$ %. For what price must she sell it in order to gain $12\frac{1}{2}$ %?

47. By selling pen points for \$ 2 a thousand, I gain $66\frac{2}{3}$ %. At what price should I sell them in order to gain 80 %?

48. I made a profit of \$ 1800 on an investment in real estate, which was a gain of $8\frac{1}{3}$ %. What was the amount of my investment?

49. By selling shoes at $33\frac{1}{3}$ % above cost, a dealer gained 95 ct. on every pair. What was the cost per pair?

50. A real estate speculator sold a house for \$ 8750. He lost $16\frac{2}{3}$ % on his investment. How much did he pay for the house?

51. A dealer bought a second-hand automobile for \$ 550. He spent \$ 185 on repairs and then sold it for \$ 1000. Find the per cent of profit.

52. A fruiterer sold an invoice of fruit for \$260, thereby gaining 4%. How much did he pay for the fruit?

53. I bought a motor boat for 10% less than it cost the builder. I sold it for \$405, thereby gaining 25% on my investment. What did it cost the builder and what did I pay for it?

54. A grocer bought a 60-pound tub of butter for \$20. He sold it at 28 ct. a pound. What per cent did he lose?

55. An invoice of kerosene oil cost \$225. The freight charge was \$54.50. The cartage was \$48.50. The insurance amounted to 5% of the invoice. For what price must the oil be sold to realize a gain of 35%?

56. A jeweler sold a watch at a loss of \$15. This was $12\frac{1}{2}\%$ of the cost. What was the cost and what was the selling price?

57. Fence posts that cost \$6.40 a hundred are marked \$10 a hundred. Before selling, the dealer deducts 10% from the marked price. What per cent does he gain?

58. A New York commission merchant sells for a packer in Maryland 3600 cans of peas at 70 ct. a dozen. What is his commission at $6\frac{3}{4}\%$?

59. A canvasser obtained 375 subscriptions to a book at \$24 a copy. His commission is 20%. What amount should he receive from his employer?

60. A real estate agent received a commission of \$225 for selling a house. What was the selling price of the house, the agent's rate of commission being $1\frac{1}{8}\%$?

61. A salesman's commissions on a 3% basis amounted to \$82.50. What was the amount of his sales?

SIMPLE INTEREST

156. Interest is the sum paid for the use of money.

Find the interest on \$200 for 1 yr. at 5 %.

5 % of \$200 = \$10, which is the *interest* on \$200 for 1 yr. at 5 %.

The **principal** is the sum of money for the use of which interest is charged.

In the above example \$200 is the **PRINCIPAL**.

The **amount** is the principal and interest added together.

\$200 loaned for 1 yr. at 5 % will amount to \$210 at the end of the year. \$210 is the **AMOUNT**.

The **rate of interest** (commonly called the **rate**) is the per cent of the principal charged for its use for one year.

In the above example 5 % is the **RATE OF INTEREST**.

NOTE. In charging interest, the year is reckoned as 12 mo. of 30 da., or 360 da.

EQUIVALENT TERMS

PERCENTAGE

Base

Rate

Percentage

Amount

INTEREST

Principal

Rate

Interest

Amount

157. The underlying principles in problems in interest are exactly the same as in the type problems in percentage. The only difference is in the introduction of *time* as a factor in the work.

Cases in Simple Interest

I. Given the *principal*, *time*, and *rate*, to find the INTEREST or AMOUNT.

II. Given the *time*, *rate*, and *interest* or *amount*, to find the PRINCIPAL.

III. Given the *principal*, *rate*, and *interest* or *amount*, to find the TIME.

IV. Given the *principal*, *time*, and *interest* or *amount*, to find the RATE.

Cases II, III, and IV are not used frequently in business. Case I is the most important.

THE DIRECT CASE IN INTEREST

158. Given the principal, time, and rate, to find the interest or amount.

ORAL EXERCISE

1. What is the interest on \$100 for 1 yr. at 6 % ? At 5 % ? At $4\frac{1}{2}$ % ?

2. What is the interest on \$1 for 1 yr. at 5 % ? For 2 yr. ? For 3 yr. ? For $\frac{1}{2}$ yr. ? For 4 yr. ?

3. What is the interest on \$50 for 1 yr. at 6 % ? For 2 yr. ? For 1 yr. at 4 % ? For 2 yr. at 4 % ?

ORAL DRILLS

	PRINCIPAL	TIME	RATE	INTEREST	AMOUNT
4.	\$240	2 yr.	5%	?	
5.	\$150	4 yr.	5%	?	
6.	\$2000	2½ yr.	6%		?
7.	\$60	3 yr.	4%		?
8.	\$75	2 yr.	5%	?	
9.	\$125	4 yr.	5%	?	?
10.	\$80	1 yr.	7%	?	
11.	\$400	1½ yr.	6%		?
12.	\$750	2 yr.	5%	?	
13.	\$300	3 yr.	8%		?
14.	\$10,000	5 yr.	6%		?
15.	\$600	8 yr.	5%	?	

The Ordinary Method

159. Find the interest on \$450 for 2 yr. 6 mo. 18 da. at 4%.

Explanation	Process
Interest on \$1 for 2 yr.	= \$.08
Interest on \$1 for 6 mo. or ½ yr.	= .02
Interest on \$1 for 18 da. or $\frac{1}{20}$ yr.	= .002
Interest on \$1 for 2yr. 6 mo. 18 da.	= \$.102
	450
	5100
	408
	<u>\$45.900</u>

SECOND METHOD

Change 2 yr. 6 mo. 18 da. to years and a decimal of a year.

Process

$$2 \text{ yr. } 6 \text{ mo. } 18 \text{ da.} = 2.55 \text{ yr.}$$

$$2.55 \times \$.04 = \$.102$$

$$450 \times \$.102 = \$ 45.90 \text{ int. } \textit{Ans.}$$

Explanation

$$\text{Interest on } \$ 1 \text{ for } 1 \text{ yr.} = \$.04$$

$$\text{Interest on } \$ 1 \text{ for } 2.55 \text{ yr.} = \$.102$$

$$\text{Interest on } \$ 450 \text{ for } 2.55 \text{ yr.} = \$ 45.90.$$

The Aliquot Part Method

160. If I pay \$6 for the use of \$100 for 1 yr., what must I pay for the use of \$100 for 3 yr.? For 5 yr.? For 7 yr.? What must I pay for the use of \$100 for $\frac{1}{2}$ yr.? For 6 mo.? For 3 mo. or $\frac{1}{4}$ yr.? For 1 mo. or $\frac{1}{12}$ yr.? For 30 da.? For 60 da.? For 10 da.? (10 da. = $\frac{1}{3}$ of 1 mo.) For 15 da.?

ORAL EXERCISES

1. Find the interest on \$300 for 2 yr. 6 mo. at 4 %.
2. Find the interest on \$200 for 3 yr. 2 mo. at 6 %.
3. Find the interest on \$400 for 1 yr. 6 mo. at 5 %.
4. Find the interest on \$500 for 9 mo. at 4 %.

WRITTEN EXERCISES

161. 1. Find the interest on \$450 for 1 yr. 8 mo. 15 da. at 6%.

Process

\$450 prin.

.06 rate

\$27.00 int. for 1 yr.

$\frac{1}{2}$ of int. for 1 yr. = 13.50 int. for 6 mo.

$\frac{1}{3}$ of int. for 6 mo. = 4.50 int. for 2 mo.

$\frac{1}{4}$ of int. for 2 mo. = 1.13 int. for $\frac{1}{2}$ mo.

\$46.13 int. for 1 yr. 8 mo. 15 da.

Explanation. Find the interest for 1 yr. at the given rate.
8 mo. = 6 mo. + 2 mo.

The interest for 6 mo. = $\frac{1}{2}$ the interest for 1 yr.

The interest for 2 mo. = $\frac{1}{3}$ the interest for 6 mo.

15 da. = $\frac{1}{2}$ of 1 mo. or $\frac{1}{4}$ of 2 mo.

The interest for 15 da. = $\frac{1}{4}$ the interest for 2 mo.

Find the sum of the interest for 1 yr. + 6 mo. + 2 mo. + 15 da. This is the required interest.

Find the interest on :

2. \$1000 for 1 yr. 4 mo. 6 da. at 6%.

3. \$720 for 4 yr. 9 mo. 24 da. at 4%.

4. \$700 for 5 yr. 6 mo. at $4\frac{1}{2}\%$.

5. \$201 for 1 yr. 90 da. at 6%.

6. \$1500 for 1 yr. 8 mo. at 5%.

7. \$2000 for 8 mo. 12 da. at 4%.

8. \$1200 for 7 mo. 15 da. at 5%.

9. \$75.80 for 3 yr. 6 mo. at 6 %.
10. \$112.50 for 4 yr. 4 mo. 24 da. at 5 %.

The Six Per Cent Method

162. At 6 % the interest on \$1 is

$$\begin{aligned} & \$.06 \quad \text{for 1 yr.} \\ & .001\frac{1}{2} \quad \text{for 1 mo.} \\ & .000\frac{1}{8} \quad \text{for 1 da} \end{aligned}$$

Find the interest on \$360 for 2 yr. 8 mo. 24 da.
at 6 %.

$$\begin{aligned} \text{Interest on \$1 for 2 yr.} &= 2 \times \$.06 = \$.12 \\ \text{Interest on \$1 for 8 mo.} &= 8 \times .005 = .04 \\ \text{Interest on \$1 for 24 da.} &= 24 \times .000\frac{1}{8} = .004 \\ \text{Interest on \$1 for given time} &= \underline{\$.164} \\ \text{Interest on \$360 for given time} &= 360 \times \$.164 \end{aligned}$$

$$\begin{array}{r} \$.164 \\ 360 \\ \hline 9840 \\ 492 \\ \hline \$59.040 \text{ Int. Ans.} \end{array}$$

Find the interest on \$1200 for 2 yr. 7 mo. at
5 % (using the 6 % method).

$$\begin{aligned} \text{Interest on \$1 for 2 yr.} &= 2 \times \$.06 = \$.12 \\ \text{Interest on \$1 for 7 mo.} &= 7 \times .005 = .035 \\ & \$.155 \qquad \qquad \underline{\$.155} \end{aligned}$$

$$\begin{array}{r} 1200 \\ \hline 6) \$186.000 \text{ Int. at 6 \%} \\ \$ 31 \text{ Int. at 1 \%} \\ 5 \\ \hline \$155 \text{ Int. at 5 \%} \end{array}$$

163. Interest at rates lower than 6 % may be found by subtracting parts of the interest at 6 % from the interest at 6 %.

Interest at rates higher than 6 % may be found by adding parts of the interest at 6 % to the interest at 6 %.

To find interest at

3 % take $\frac{1}{2}$ of the interest at 6 %.

4 % subtract $\frac{1}{3}$ of the interest at 6 %.

$4\frac{1}{2}$ % subtract $\frac{1}{4}$ of the interest at 6 %.

5 % subtract $\frac{1}{6}$ of the interest at 6 %.

$5\frac{1}{2}$ % subtract $\frac{1}{12}$ of the interest at 6 %.

7 % add $\frac{1}{6}$ of the interest at 6 %.

$7\frac{1}{2}$ % add $\frac{1}{4}$ of the interest at 6 %.

8 % add $\frac{1}{3}$ of the interest at 6 %.

ORAL EXERCISES

164. 1. Find the interest on \$60 for 8 mo. at 6 %.

2. Find the interest on the same sum, for the same time at $4\frac{1}{2}$ %. At 7 %. At 8 %. At 4 %.

Find the interest on :

PRINCIPAL	TIME	RATE %
3. \$150	1 yr. 6 mo.	6
4. \$150	1 yr. 6 mo.	5
5. \$150	1 yr. 6 mo.	9
6. \$300	2 yr. 3 mo.	6
7. \$300	2 yr. 3 mo.	8

	PRINCIPAL	TIME	RATE %
8.	\$ 300	2 yr. 3 mo.	7
9.	\$ 40	1 yr. 3 mo.	6
10.	\$ 40	1 yr. 3 mo.	4½

WRITTEN EXERCISES

165. Find the sum indicated by ? in the following examples :

	PRINCIPAL	TIME	RATE %	INTEREST	AMOUNT
1.	\$ 280	2 yr. 7 mo. 15 da.	6	?	
2.	\$ 280	2 yr. 7 mo. 15 da.	4½	. . .	?
3.	\$ 280	2 yr. 7 mo. 15 da.	7	?	
4.	\$ 280	2 yr. 7 mo. 15 da.	4	?	
5.	\$ 264.80	1 yr. 8 mo.	6	?	
6.	\$ 264.80	1 yr. 8 mo.	4	. . .	?
7.	\$ 264.80	1 yr. 8 mo.	5	?	
8.	\$ 264.80	1 yr. 8 mo.	3	. . .	?
9.	\$ 264.80	1 yr. 8 mo.	4½	?	
10.	\$ 2000	7 yr. 6 mo.	6	?	
11.	\$ 2000	7 yr. 6 mo.	7	?	
12.	\$ 2000	7 yr. 6 mo.	9	. . .	?

WRITTEN PROBLEMS IN SIMPLE INTEREST

166. 1. A man has \$40,000 invested, so that 40% of it yields an interest of 4% and the rest yields 5%. Find the total amount of interest he will receive on both investments in 2 yr. and 9 mo.

2. A college has an endowment fund of \$750,000. What monthly income does it receive

from this fund, if it is invested so as to yield interest at 5 % per annum ?

3. A man places a \$ 20,000 mortgage at $4\frac{1}{2}$ % upon his property. Interest is due on Jan. 1. Beginning with April 1, what sum must he lay aside each month, so as to have the full interest on hand before payment is due ?

4. \$ 5000 loaned on March 11, 1906, amounted to what sum on Feb. 6, 1908, money being at 5 % ?

5. A man borrowed \$ 40,000 at an annual interest of 4 %. He invested it in property which yielded an interest of $5\frac{3}{4}$ %. What did he gain in 2 yr. 6 mo., by this transaction ? (Use direct method.)

6. A note of \$ 450, dated April 28, 1910, was paid May 31, 1911, with interest at 7 %. What was the amount paid ?

7. What interest must I pay semi-annually to the holder of a \$ 7500 mortgage on my house, the rate being $4\frac{1}{2}$ % ?

8. The court decided that the defendant in a lawsuit must pay the plaintiff the sum of \$ 36,120, with interest at 6 % from Aug. 18, 1905, to Dec. 1, 1910. What was the amount to be paid ?

9. A note for \$ 120.50, dated Feb. 10, 1910, was due June 15, 1910 ; what was the amount due, the note bearing interest at 5 % ?

10. The trustees of an estate paid to the heirs on Jan. 1, 1911, the proceeds of a sale of bonds for \$26,750 made on Sept. 10, 1905, with interest at $3\frac{1}{2}\%$ to the date of payment. How much was paid to the heirs?

11. A friend borrowed \$250 from me on Dec. 15. He promised to repay me on Sept. 1 following, with interest at the rate of 7% per annum. How much did he owe me on the due date?

12. Find the interest on a note for \$67.50, dated March 25, 1908, and payable Oct. 10, 1910, the rate being $5\frac{1}{2}\%$.

13. How much is due on a debt of \$650 on which interest is chargeable for 1 yr. 3 mo. at $4\frac{3}{4}\%$?

INTEREST. SUPPLEMENTARY CHAPTER *

The Direct Case in Interest

167. Given the principal, time and rate, to find the interest or amount.

The 36 Per Cent Method

At 6%, the interest on \$1 is

\$.06	for 1 yr.
.005	for 1 mo.
.000 $\frac{1}{6}$	for 1 da.

* The object of this chapter is to furnish additional material for schools that study the derived cases of interest.

At 36 %, the interest on \$1 is

\$.36	for 1 yr.
.03	for 1 mo.
.001	for 1 da.

Therefore we have the following

PRINCIPLE. The interest on any principal for 1 da at 36 % equals .001 of that principal.

Find the interest on \$ 375 for 2 yr. 7 mo. 16 da. at 4 %.

Process

$$\begin{array}{r}
 \text{da.} \\
 2 \text{ yr.} = 720 \\
 7 \text{ mo.} = 210 \\
 16 \text{ da.} = 16 \\
 \hline
 946 \text{ da.} \\
 \\
 \$.375 \\
 \quad 946 \\
 \hline
 2 \ 250 \\
 15 \ 00 \\
 337 \ 5 \\
 \hline
 9) \$ 354.750 \text{ int. at } 36 \% \\
 \quad \$ 39.42 \text{ int. at } 4 \% \quad \text{Ans.}
 \end{array}$$

Explanation

Change 2 yr. 7 mo. 16 da. to days. The interest on \$375 for 1 da. at 36 % = \$.375. The interest on \$375 for 946 da. at 36 % = $946 \times \$.375 = \$ 354.75$. \$354.75 = the interest at 36 %; at 4 % the interest will be $\frac{1}{9}$ of \$354.75 or \$39.42.

Rule for finding interest at 36 %.

1. Point off three places in the principal.
2. Multiply the result by the number of days for which the money is loaned.

163. From the interest on any principal at 36 %, the interest at other rates may be found as follows:

To find interest at

3 % take $\frac{1}{12}$ of the interest at 36 %.

4 % take $\frac{1}{9}$ of the interest at 36 %.

$4\frac{1}{2}$ % take $\frac{1}{8}$ of the interest at 36 %.

5 % { take $\frac{1}{6}$ of the interest at 36 % and then
subtract $\frac{1}{6}$ of that number; or divide the
interest at 36 % by 36, and multiply
the quotient by 5.

7 % { take $\frac{1}{6}$ of the interest at 36 % and then
add $\frac{1}{6}$ of that number; or divide the in-
terest at 36 % by 36, and multiply the
quotient by 7.

8 % { take $\frac{1}{9}$ of the interest at 36 % and then
multiply by 2.

ORAL EXERCISE

169. Work all the examples under Art. 165 by the 36 % method.

WRITTEN EXERCISE

170. Work all the examples under Art. 166 by the 36 % method.

THE DERIVED CASES IN INTEREST

171. Given the time, rate, and interest or amount, to find the principal.

(A). What sum of money placed at 4 % int. for $7\frac{1}{2}$ yr. will yield a total interest of \$1200 ?

Process

$$\begin{array}{r}
 \$.04 \\
 7\frac{1}{2} \\
 \hline
 \$.30 \\
 .30 \overline{)1200 \times 0.} \\
 \hline
 4000.
 \end{array}$$

Explanation. The interest on \$1 for $7\frac{1}{2}$ yr. at 4 % = \$.30.

Therefore, the number of dollars required to yield \$1200 interest will equal $1200 \div .30$ or 4000.

Ans. \$4000.

(B). What sum of money placed at 5 % int. for 3 yr. 6 mo. will amount to \$2350 ?

Process

$$\begin{array}{r}
 \$.05 \\
 3\frac{1}{2} \\
 \hline
 \$.17\frac{1}{2} \\
 2000. \\
 1.175 \overline{)2350 \times 000.} \\
 \hline
 2350
 \end{array}$$

Explanation. The interest on \$1 for 3 yr. 6 mo. at 5 % equals \$.17 $\frac{1}{2}$; the amount of \$1 for 3 yr. 6 mo. at 5 % equals \$1.17 $\frac{1}{2}$.

Therefore, the number of dollars required to yield an amount of \$2350 equals $2350 \div 1.175$ or 2000.

Ans. \$2000.

RULE. Given the time, rate, and interest, to find the principal.

Divide the given interest by the interest on one dollar for the given time and rate.

RULE. Given the time, rate, and amount, to find the principal.

Divide the given amount by the amount of one dollar for the given time and rate.

TO FIND THE TIME

172. Given the principal, rate, and interest or amount, to find the time.

(A). How long must \$150 be placed at interest at 6 % to yield \$45 ?

Process

$$\begin{array}{r}
 \$150 \\
 \underline{.06} \\
 \$9.00 \text{ int. for 1 yr. at 6 \% .} \\
 9 \overline{)45} \\
 \underline{5} \text{ yr.}
 \end{array}$$

Explanation. At 6 %, \$150 yields \$9 interest in one year. Therefore the number of years required to yield 45 equals the number of 9's in 45.

Ans. 5 yr.

(B). How long must \$240 be placed at interest at 5 % to amount to \$330 ?

Process

$$\begin{array}{r}
 \text{Amt. } \$330 \\
 \text{Prin. } \$240 \\
 \hline
 \text{Int. } \$90 \\
 \$240 \\
 \underline{.05} \\
 \$12.00 \text{ int. for 1 yr.} \\
 12 \overline{)90} \\
 \underline{7} \frac{1}{2} \text{ yr.}
 \end{array}$$

Explanation. Since \$330 is the amount, and \$240 is the principal, the interest is \$330 — \$240, or \$90.

At 5 % \$240 yields \$12 interest in one year. Therefore the number of years required to yield \$90 equals the number of 12's in 90, or $90 \div 12$.

Ans. $7\frac{1}{2}$ yr.

RULE. Given the principal, rate, and interest or amount, to find the time.

Divide the given interest by the interest on the given principal for one year at the given rate.

TO FIND THE RATE

173. Given the principal, time, and interest or amount, to find the rate.

(A). At what rate will \$160 yield an interest of \$24 in 3 yr.?

Process

Int. on \$1 for 1 yr. at 1% = \$.01

Int. on \$1 for 3 yr. at 1% = .03

Int. on \$160 for 3 yr. at 1% = 4.80

$$\begin{array}{r} 5 \\ 4.80 \overline{)24.00} \\ \underline{240} \end{array} \quad \text{Ans. 5 yr.}$$

Explanation. The interest on \$160 for 3 yr. at 1% = \$4.80. Therefore the rate per cent needed to yield \$24 interest equals $24 \div 4.80$. Ans. 5 %.

(B). At what rate will \$72 amount to \$86.40 in 2 yr. 6 mo.?

Process

Int. on \$1 for $2\frac{1}{2}$ yr. at 1% = \$.02 $\frac{1}{2}$

Int. on \$72 for $2\frac{1}{2}$ yr. at 1% = 1.80

$$\begin{array}{r} 8 \\ 18.0 \overline{)14.40} \\ \underline{144} \end{array} \quad \text{Ans. 8 \% .}$$

Explanation. Find the interest by subtracting the principal from the amount. $\$86.40 - \$72 = \$14.40$ interest. Continue as in Example (A).

RULE. Given the principal, interest or amount, and time, to find the rate.

Divide the given interest by the interest on the given principal for the given time at one per cent.

WRITTEN DRILL

174. Find the number represented by ?

	PRIN.	TIME	RATE %	INT.	AMOUNT
1.	\$150	yr. 2 mo. 6	?	\$15.00	
2.	\$250	?	5	\$37.50	
3.	\$7000	yr. 2 mo. 4	6	?	
4.	?	yr. 3	4	\$224
5.	\$840	yr. 3 mo. 6	?	\$987
6.	\$96.40	?	5	\$14.66	
7.	?	yr. 16 $\frac{1}{2}$	6	\$1200
8.	\$40000	mo. 7 d. 15	?	\$1000	
9.	\$672	?	4	\$13.44	
10.	\$1440	mo. 10 d. 20	6	?
11.	?	yr. 10	3 $\frac{1}{2}$	\$10.50	
12.	\$8000	?	4 $\frac{1}{2}$	\$9200
13.	\$96	yr. 2 mo. 2	?	\$12.48	
14.	\$480	mo. 3 d. 18	4 $\frac{1}{2}$?	
15.	?	yr. 5	6	\$93	
16.	\$1200	?	5	\$1620
17.	\$2500	yr. 7 $\frac{1}{2}$?	\$750	
18.	?	yr. 12 mo. 6	8	\$2000	
19.	\$1728	5	4	?	



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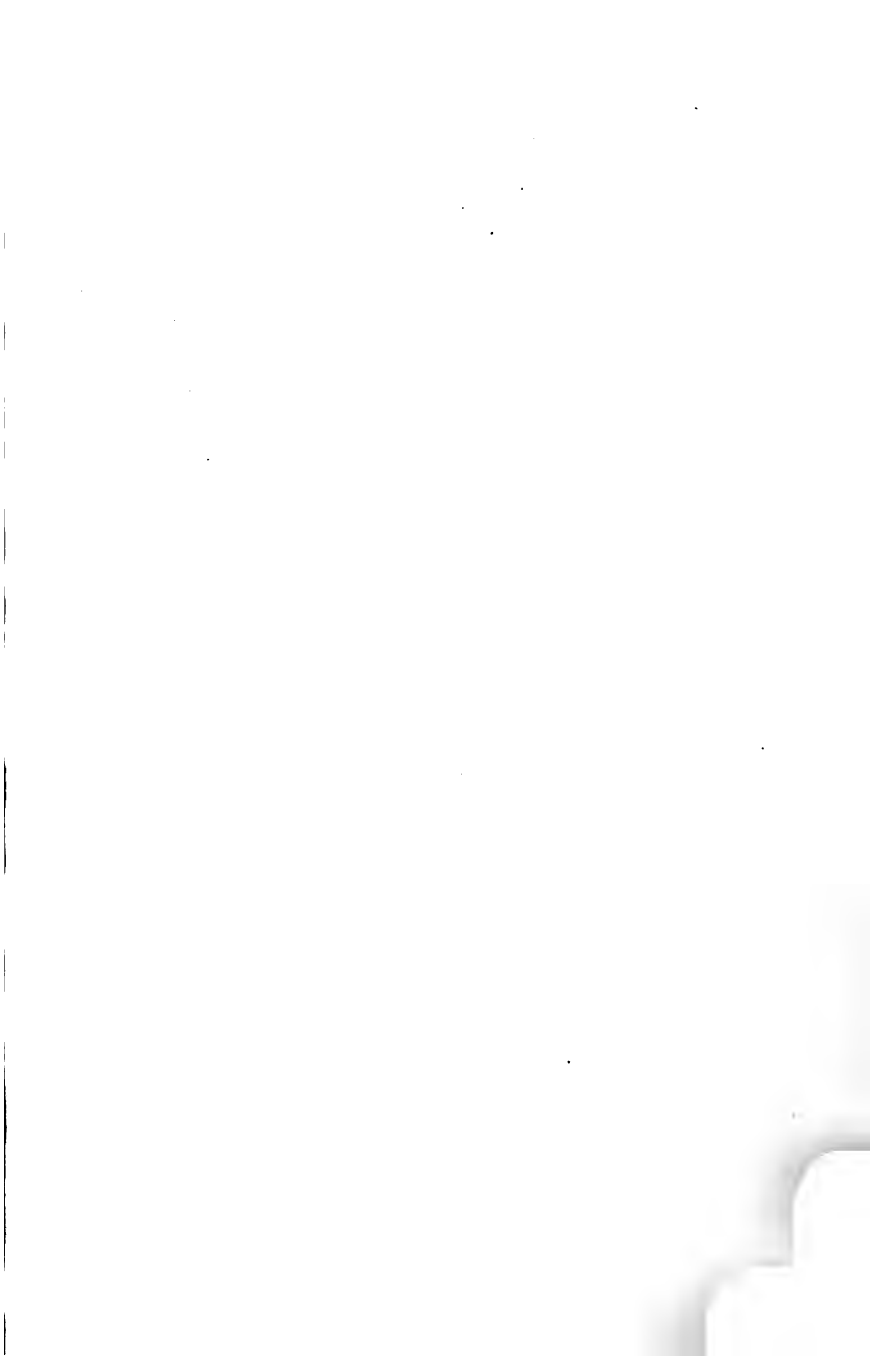
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